Numerical Model for Biaxial Loading of Reinforced Concrete Strengthened with FRP

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ABSTRACT: The application of shear walls as a primary earthquake resistant mechanism in structural design is traditionally accepted and used over the last decades. However, very often, older structures having shear walls no longer comply with the contemporary standards and codes which raise the need for their strengthening and retrofit. Many strengthening measures and techniques have been designed and used. One of the newest and most promising is the use of externally bonded FRP strips and sheets. Intensive research has started in order to design the most economic and efficient technique for application of these materials. Different modeling approaches have been employed in these designs. This paper presents the attempt to formulate a new material model that could be used to model FRP strengthened RC member and its implementation into ANSYS. The model results are tested against the available experimental data in order to verify its correctness and practical usability.

1 INTRODUCTION

The conventional earthquake resistant design of reinforced concrete structures advises use of shear walls as effective way to add earthquake resistance to the reinforced concrete frames. A problem arises with structures erected decades ago following design rules which are by today's standards obsolete, inadequate and inefficient. Major earthquake events from around the world have shown the design deficiencies of these structures by inducing extensive damages in the structural members. Many of the old shear wall buildings are at risk of suffering damages from a major earthquake mostly due to their insufficient in-plane stiffness, flexural and shear strengths and ductility owing to the older design codes which didn't adequately estimate the demands that major earthquakes impose on the structures. This problem is ever increasing as the existing structures are getting older and their members gradually deteriorate.

Many different methods of seismic strengthening and repair of shear wall structures have been developed and tested in the last thirty years. Recently, state-of-the-art strengthening and retrofit techniques increasingly utilize externally bonded fibre reinforced polymer (FRP) composites, which offer unique properties in terms of strength, lightness, chemical resistance, and ease of application. Such techniques are most attractive for their fast execution and low labour costs.

Only recently have researchers attempted to simulate the behavior of reinforced concrete strengthened with FRP composites using the finite element method. The majority of the studies that included numerical modeling of FRP strengthened RC members with FEM use element overlaying, where one-, two- or even three dimensional elements (solid or layered) that represent the FRP material are superimposed over the concrete elements, either with (ex. Khomwan and Foster, 2004; Wong and Vecchio, 2003) or without (ex. Kheyroddin and Naderpour, 2008) interface elements that represent the influence of the adhesive material or the bond between the FRP and the concrete.

A different approach is presented in this paper. An attempt is made to formulate a new material model which will simplify the modeling of FRP strengthened reinforced concrete members. The newly formulated material model is implemented into ANSYS and tested using available experimental data.

2 MODEL FORMULATION

In the analysis of RC structures plane stress problems make up a large majority of practical cases. Therefore, the numerical model presented here is based on the inelastic model for cyclic biaxial loading of reinforced concrete of Darwin and Pecknold (1974) which was designed to be used for such type of structures (shear walls, beams, slabs, shear panels, shells, reactor containment vessels).

2.1 Concrete

The concrete is treated as incrementally linear, elastic material, which means that during each load increment the material is assumed to behave elastically. It is also considered to exhibit stress-induced orthotropic material behavior. The constitutive relationship for incrementally linear orthotropic material with reference to the principal axes of orthotropy can be written as:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \frac{1}{1-v^2} \begin{bmatrix} E_1 & v\sqrt{E_1 E_2} & 0 \\ v\sqrt{E_1 E_2} & E_2 & 0 \\ 0 & 0 & (1-v^2)G \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} = D_C \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \quad (1)$$

where $d\sigma_i$ and $d\varepsilon_i$ are the stress and strain increments, E_1 and E_2 are initial concrete stiffness modules in principal directions, $v = v_1 \cdot v_2$ is the "equivalent" poison ratio, $G = \frac{1}{4(1-v^2)}(E_1 + E_2 - 2v\sqrt{E_1 E_2})$ is the shear modulus and D_C is the concrete constitutive matrix in the principle directions. Before it can be used in the finite element procedure, the concrete constitutive matrix is transformed to global coordinates using:

$$D'_C = T^T D_C T \quad (2)$$

where T is the strain transformation matrix (Cook, 1974). At the moment when the principle tensile stress exceeds the concrete tensile strength a "crack" forms perpendicular to the principle stress direction. This is modeled by reducing the values of E and v to zero. This has an effect of creating a "smeared" rather than discrete crack. The constitutive equation for the cracked concrete then takes the form:

$$\begin{Bmatrix} d\sigma_1 \\ d\sigma_2 \\ d\tau_{12} \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & \frac{E_2}{4} \end{bmatrix} \begin{Bmatrix} d\varepsilon_1 \\ d\varepsilon_2 \\ d\gamma_{12} \end{Bmatrix} \quad (3)$$
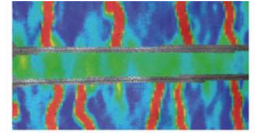
If the tensile strength in the other principle direction is exceeded then a second crack occurs and the constitutive matrix is then reduced to $D_C = [0]$. In order to keep track of the material degradation, the concept of "equivalent uniaxial strain" is used. It allows derivation of the actual biaxial stress-strain curves from uniaxial curves. The equation suggested by Saenz (1964) is often used for this purpose:

$$\sigma_i = \frac{\varepsilon_{ui} \cdot E_0}{1 + \left(\frac{E_0}{E_s} - 2\right)\frac{\varepsilon_{ui}}{\varepsilon_{ci}} + \left(\frac{\varepsilon_{ui}}{\varepsilon_{ci}}\right)^2} \tag{4}$$

where E_0 is the tangent modulus of elasticity at zero stress, E_s is the secant modulus at the point of maximum compressive stress (σ_{ci}), and ε_{ci} is the equivalent uniaxial strain at maximum compressive stress, Figure 1. The concrete biaxial strength envelope suggested by Kupfer and Gerstle (1973), (Figure 2.) is used to determine the value of σ_{ci}.
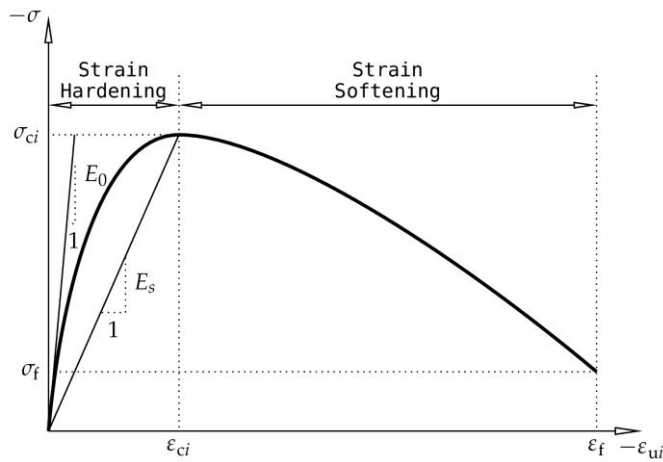

Figure 1. Equivalent Uniaxial Stress-Strain Curve in Compression, Hu and Schnobrich (1989).
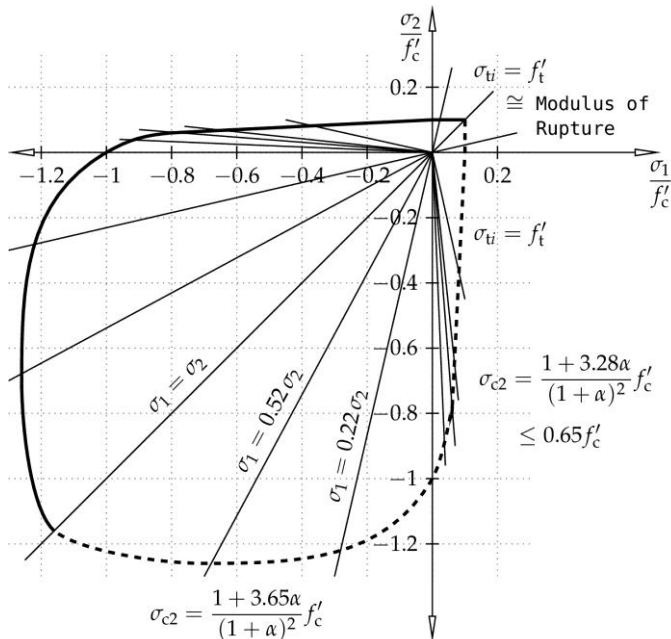

Figure 2. Analytical Biaxial Strength Envelope, Kupfer and Gerstle (1973).

2.2 Reinforcing Steel

Generally, the reinforcing steel can be modeled as discrete or distributed. The model presented here considers the reinforcing steel to be distributed, or "smeared", throughout the concrete. A simple, bilinear model with strain hardening is adopted for the stress-strain behavior of the steel. The constitutive matrix of the steel defined in the steel direction is

$$D_S = p_S \begin{bmatrix} E_{Steel} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

with E_{Steel} the tangent stiffness of the steel and p_S the reinforcing ratio. Depending on the stress level in the steel, E_{Steel} can be either equal to the initial steel stiffness E_S or reduced by a strain hardening stiffness ratio (δ), see Figure 3. Before using it in the composite material matrix, D_S is transformed to the global coordinates using the strain transformation matrix (T).

2.3 FRP strengthening

The influence of the FRP strengthening is accounted for in the same fashion as the reinforcing steel. The material is treated as distributed, or "smeared" throughout the concrete. Its material behavior is assumed to be elastic-brittle, having abrupt failure after reaching its maximal strength (Figure 3). It is also capable of transmitting only tension stresses. The constitutive matrix of the FRP defined in the direction of the FRP fibers is therefore:

$$D_F = p_F \begin{bmatrix} E_F & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

with E_F the tangent stiffness of the FRP and p_F the "strengthening" ratio. Before using it in the composite material matrix D_F must also transformed to the global coordinates using the strain transformation matrix (T).
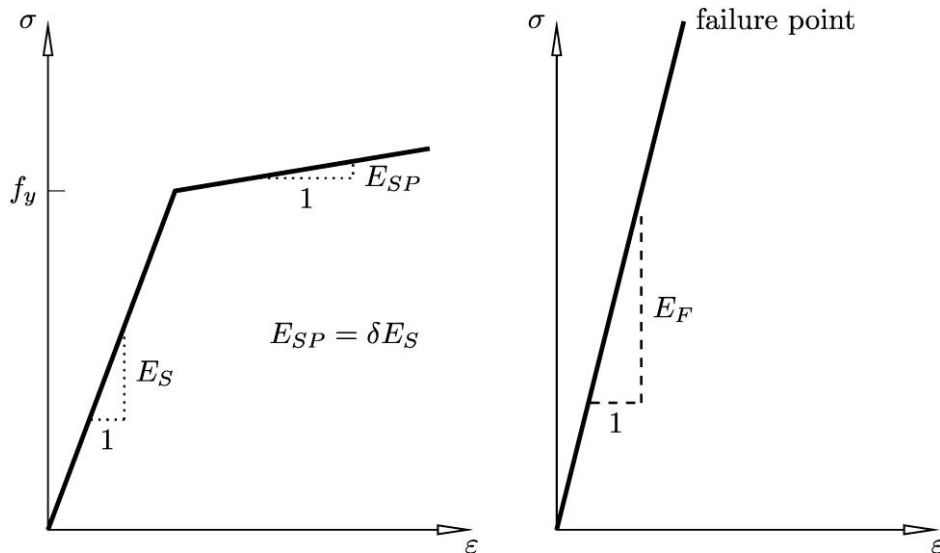

Figure 3. Stress-strain curves of reinforcing steel and FRP used in the model

2.4   Composite Material Matrix

After defining the constitutive matrices of the constituent materials, the constitutive matrix of the composite material in the global coordinates is obtained by their summation:

$$D' = D'_C + \sum_{i=1}^{n} D'_{S,i} + \sum_{i=1}^{m} D'_{F,i} \tag{7}$$

where D', D'_C, D'_S and D'_F are the constitutive matrices if the composite material, concrete, steel and FRP in global coordinates, respectively, n is the number of different reinforcing steels and m is the number of different FRPs used for strengthening.

3   VERIFICATION

The material model briefly described in the previous section was coded and implemented into ANSYS in order to test its correctness and usability by comparing the results from numerical analyses with the available experimental data from the literature. The results of the analysis of three-large scale RC beams tested by Wong (2001) are presented here. These were also analysed by Wong and Vecchio (2003). The beams were designed with only tension reinforcement. No shear steel reinforcement was used. Instead, CFRP strips were glued to the side surfaces to act as shear strengthening. The geometry of the three beam specimens is shown in Figure 4. The CFRP fabric used for strengthening was composed of graphite fibers oriented in the longitudinal direction and Kevlar 49 weft in the perpendicular direction. The material was tested to obtain its material properties. Tensile strength of 1090 MPa and ultimate strain of 0.011 was recorded. Strips of this material with a width of 200 mm were bonded on the side surfaces (not wrapped around the beam) at a central distance of 300 mm between each other. The beams were tested under monotonic three-point loading until failure.
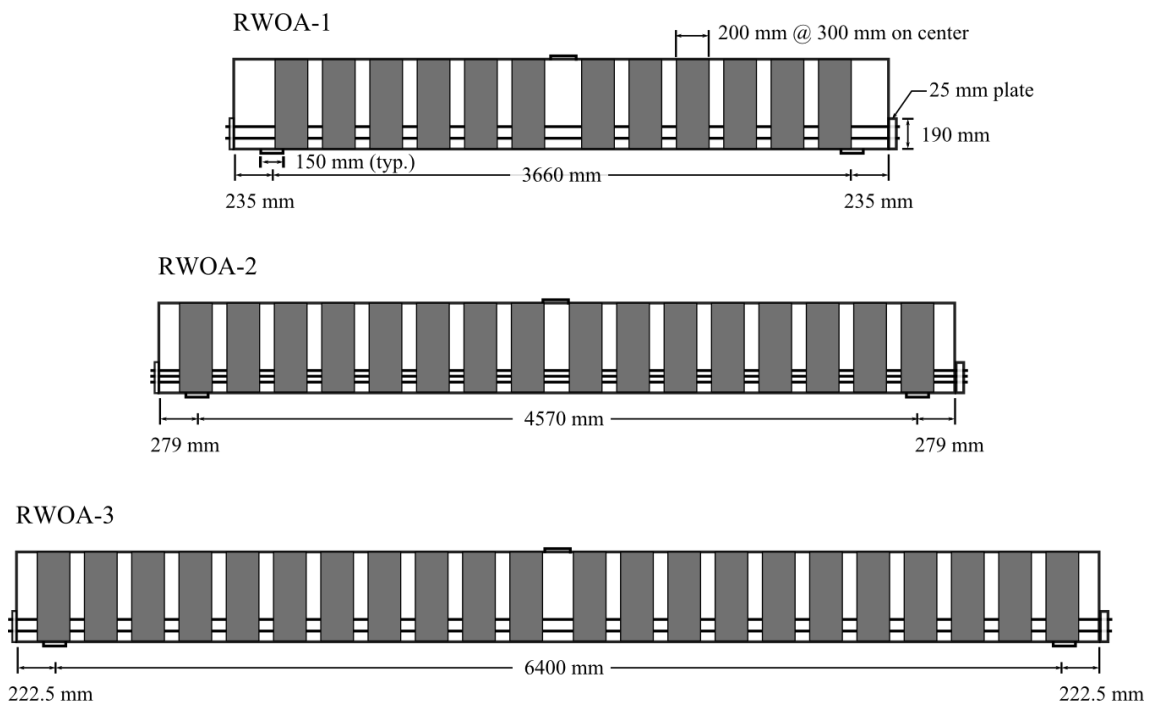

Figure 4. Elevation views of RWOA beams, Wong (2001)

In the FEM analyses performed by Wong and Vecchio (2003), 2D elements were used to represent the concrete. The elements were double-noded with one set of nodes used for the concrete elements, while the second set was used to attach the truss elements that represented the FRP. Then the coincident nodes were connected by contact or link elements representing the bond.

The finite element model used here was build using 4-node quadrilateral Plane182 elements. Only half of the each test specimen was modeled making use of its symmetry (Figure 5). To properly model the longitudinal steel reinforcement, the beams were divided into two parts - upper and lower part, with the lower part being of height of 130 mm and containing the smeared longitudinal reinforcement. The material properties of the two parts for each of the beams are presented in Table 1. The available experimental data was used to calibrate the models. The resulting load-deflection curves are shown in Figure 6. They are compared with the recorded experimental data as well as with the results of the FEM analysis performed by Wong and Vecchio (2003). It can be seen that the obtained results closely follow the experimental curves especially in the deflection range up to the steel yielding point. The failure in the models occurred due to concrete crushing at the top point in the symmetry axes, i.e. the location where the load is applied. The performed analyses showed different results in predicting the point of failure which was mostly influenced by the finite element and load step sizes.
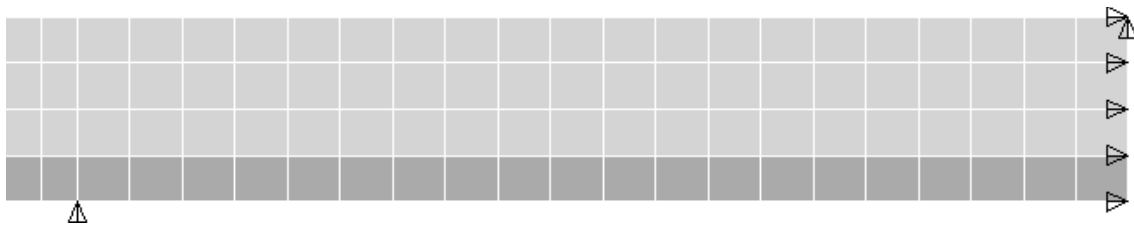

Figure 5. FEM mesh model of RWOA beams in ANSYS

Table 1. Material Parameters for the RWOA Beams used in the Analysis

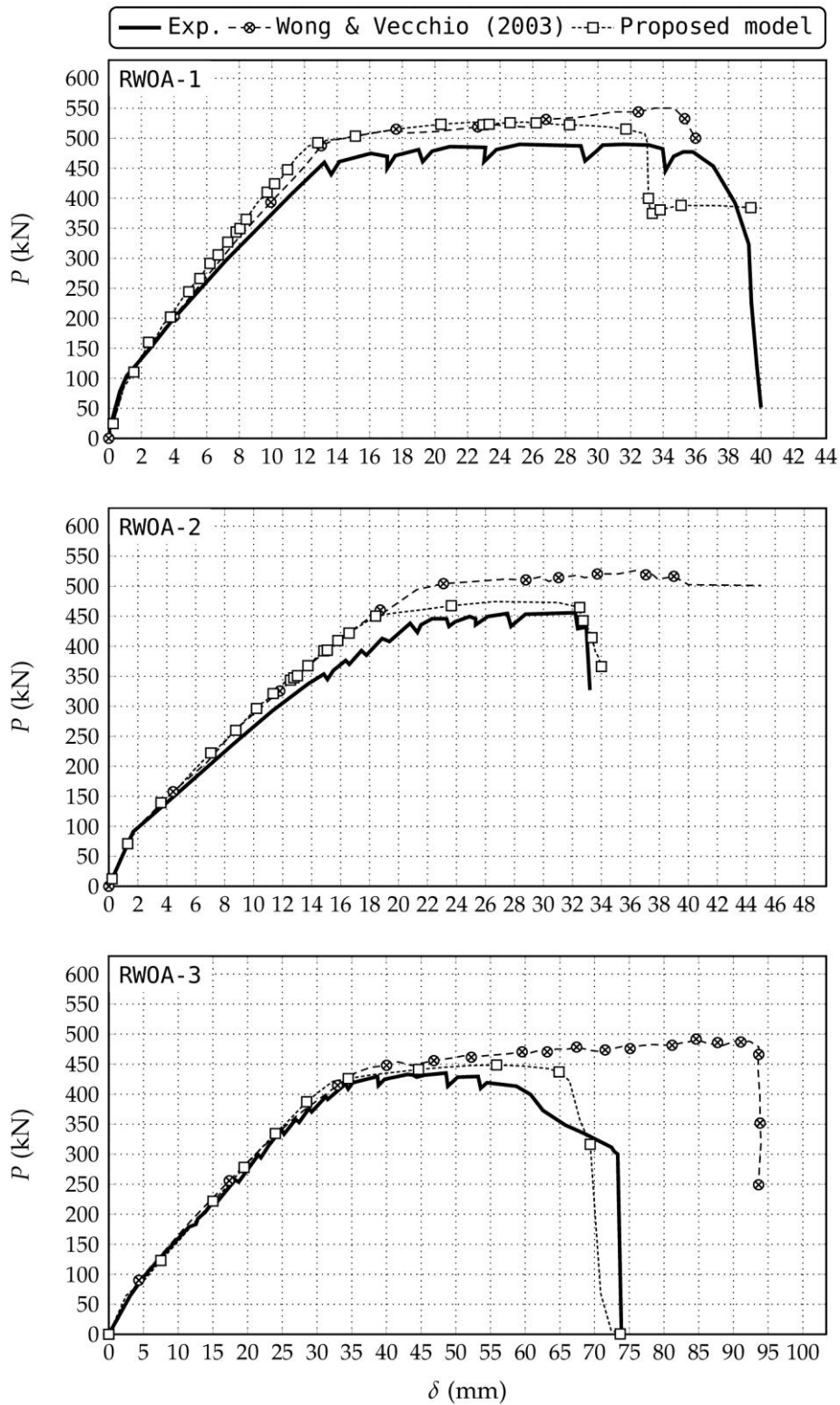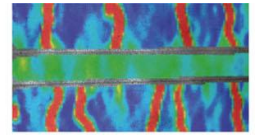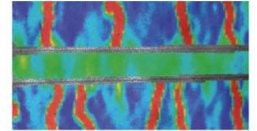|	Concrete					Steel				FRP		
	f_c' MPa	E_0 GPa	f_t MPa	ε_{cu} %	ν	f_y MPa	E_S GPa	δ %	p_S %	E_F GPa	ε_F %	p_F %
RWOA 1												
upper	23	18	4	-0.35	0.2					100	1.1	0.2
lower	23	18	4	-0.35	0.2	430	200	1	5.6	100	1.1	0.2
RWOA 2												
upper	26	20	4	-0.35	0.2					100	1.1	0.2
lower	26	20	4	-0.35	0.2	400	200	1	7.3	100	1.1	0.2
RWOA 3												
upper	44	25	4	-0.35	0.2					100	1.1	0.2
lower	44	25	4	-0.35	0.2	400	200	1	8.9	100	1.1	0.2

Figure 6. Load versus mid span deflection for RWOA beams, Wong (2001)

4   SUMMARY AND CONCLUSIONS

The paper presents an attempt to formulate material model which will correctly simulate the behaviour of reinforced concrete members in plane stress strengthened with FRP materials. The presented model builds up on the concepts of an earlier reinforced concrete model of Darwin and Pecknold. It uses the uniaxial strain approach in modeling of biaxially loaded reinforced concrete and the distributed approach of modelling the cracking behaviour and the reinforcing steel.

The proposed model is subsequently implemented into the code of the general finite element method program ANSYS as a user material model in order to test its results against the available experimental data. Analyses of three RC beams strengthened by externally bonded FRP wraps on their sides were performed and presented herein. The results are compared against the experimentally obtained data as well as against the numerical results from another finite element analysis performed by other authors employing more traditional approach into finite element modelling if such problems. Based on the presented results, it can be concluded that the proposed model is able to correctly simulate the behaviour of the RC beams strengthened with FRP. Its ANSYS implementation enables its use in both research and practical purposes, facilitating the further research in this field as well as the practical applications in the construction industry.

5   REFERENCES

ANSYS Inc., ANSYS®Multiphysics, Release 9.0

Cook, R. D. (1974). Concepts and applications of finite element analysis. New York: John Wiley & Sons.

Darwin, D. and Pecknold, D.A.W. (1974). Inelastic Model for Cyclic Loading of Reinforced Concrete, Civil Engineering Studies SRS-409, University of Illinois, Urbana Champaign, Illinois, 169 pages.

Hu, H.-T. and Schnobrich, W. C. (1989). Constitutive modeling of concrete by using nonassociated plasticity, *Journal of Materials in Civil Engineering*, 1, 199–216.

Kheyroddin, A. & Naderpour, H. (2008). Nonlinear finite element analysis of composite RC shear walls, *Iranian Journal of Science & Technology, Transaction B, Engineering*, 32(B2), 79–89.

Khomwan, N. and Foster, S. J. (2004). Finite Element Modelling of FRP Strengthened Beams and Walls. Technical Report UNICIV Report R-432, The University of New South Wales, School of Civil and Environmental Engineering, Kensington, Sydney 2052 Australia, 68 pages.

Kupfer, H., Hilsdorf, H.K. and Rusch (1969). Behavior of Concrete Under Biaxial Stresses, *ACI Journal Proceedings*, 656-666.

Kupfer, H. and Gerstle, K.H. (1973). Behavior of Concrete Under Biaxial Stresses, *Journal of Engineering Mechanics Division*, 852-866.

Saenz, L. (1964). Discussion of "Equation for Stress-Strain Curve of Concrete" by Desayi and Krishman, *ACI Journal*, 1229-1235

Wong, R. S. Y. (2001). Towards modelling of reinforced concrete members with externally-bonded fibre reinforced polymer (FRP) composites. Master's thesis, University of Toronto, Toronto, Canada.

Wong, R.S.Y. and Vecchio, F.J. (2003). Towards modeling of reinforced concrete members with externally bonded fiber-reinforced polymer composites. *ACI Structural Journal* **100:1**,47-55.