

Analytical modeling of RC beams strengthened with prestressed NSM-CFRP strips subjected to freeze-thaw exposure

Hamid Y. Omran¹ and Raafat El-Hacha²

¹ PhD Candidate, Department of Civil Engineering, University of Calgary, Calgary, Canada

² Associate Professor, Department of Civil Engineering, University of Calgary, Calgary, Canada

ABSTRACT: This paper demonstrates a nonlinear analytical model for predicting the load-deflection responses of Reinforced Concrete (RC) beams strengthened with prestressed or non-prestressed Near-Surface Mounted (NSM) Carbon Fiber Reinforced Polymer (CFRP) strips subjected to freeze-thaw cycling exposure. The load-deflection responses of the beams tested under four-point bending configuration were generated using numerical integration of the curvature along the length of the beam. A freeze-thaw exposed concrete stress-strain curve was generated and assigned to the model. Furthermore, the model has the capabilities of assigning the elasto-plastic material properties for the compression and tension steel reinforcements, the linear-elastic material properties for the CFRP reinforcements, and the partial prestressing length of the NSM-CFRP reinforcements along the length of the beam. The predicted results from the analytical model were validated with the experimental responses confirming superior accuracy of the developed model.

1 INTRODUCTION

The new generation of materials produced over the last two decades has altered the conventional rehabilitation/strengthening methods of structures. Nowadays, Carbon Fiber Reinforced Polymers (CFRPs) play a major role in upgrading the Reinforced Concrete (RC) members and even steel members. The prestressed Near-Surface Mounted (NSM) CFRP method is one of the strengthening techniques that has been used recently for improving the flexural performance of concrete members in which the CFRP rebars or strips are tensioned and mounted inside a precut groove on the tension side of the concrete member filled with epoxy adhesive. In earlier researches (Nordin & Täljsten, 2006; De Lorenzis & Teng, 2007; Badawi & Soudki, 2009), prestressing of the NSM-CFRP reinforcements was performed against both ends of the RC beam or against an independent steel reaction frame that made the prestressed NSM method unsuitable for real projects. The practical issue of the prestressed NSM method was solved with the development of an innovative mechanical anchorage system enabling prestressing the NSM CFRP strips or rebars against the concrete beam itself (Gaafar, 2007; El-Hacha & Gaafar, 2011). So far, the performance of the NSM-CFRP strengthened beams employing the practical prestressing system have been studied under static and fatigue loading, and freeze-thaw cycling exposure by a few researchers (Gaafar, 2007; El-Hacha & Gaafar, 2011; Oudah, 2011; Oudah & El-Hacha, 2012a, 2012b, and 2012c; Omran & El-Hacha, 2012a).

On the other hand, the analytical studies on the NSM-CFRP strengthened concrete members need to be pursued in the evolution of this strengthening system parallel to the experimental investigations. Therefore, the main objective of this paper is to develop an analytical model to simulate the load-deflection response of the prestressed or non-prestressed NSM-CFRP

strengthened beams subjected to freeze-thaw cycling exposure. The model accounts for the exposed freeze-thaw concrete material, compression and tension steel reinforcements, partial prestressing length of NSM-CFRP along the length of the beam (since the beams might not be strengthened for entire length with NSM-CFRP), type of loading (four-point bending configuration), and mode of failure. Also, the developed model is verified with experimental test results.

2 EXPERIMENTAL PROGRAM OVERVIEW

Five RC beams were tested including one un-strengthened control beam, one strengthened beam with non-prestressed NSM-CFRP strips, and three strengthened beams with prestressed NSM-CFRP strips. Details of the strengthened beams are presented in Figures 1 and 2. The beams were simply supported having 5000 mm span length and rectangular cross-section of 200×400 mm. Each beam was strengthened with 2×16 mm strips glued together from the side and mounted in one groove precut on the tension side of the beam. Various prestressing levels of 0%, 16.5%, 32%, and 47% of the ultimate tensile strain of the CFRP strips were enforced to the NSM CFRP strips (the prestrain values are presented in Table 1). The beams were loaded after strengthening up to 1.2 times the analytical cracking load for each beam, to accelerate the effects of freeze-thaw exposure on the specimens, then, exposed to 500 freeze-thaw cycles. Each cycle, accomplished in 8hrs, includes the lower temperature bound of -34°C and the upper temperature bound of +34°C with a relative humidity of 75% for temperature above +20°C. More details about the experimental program can be found in Omran & El-Hacha (2012a). A summary of the test results is provided in Table 1.

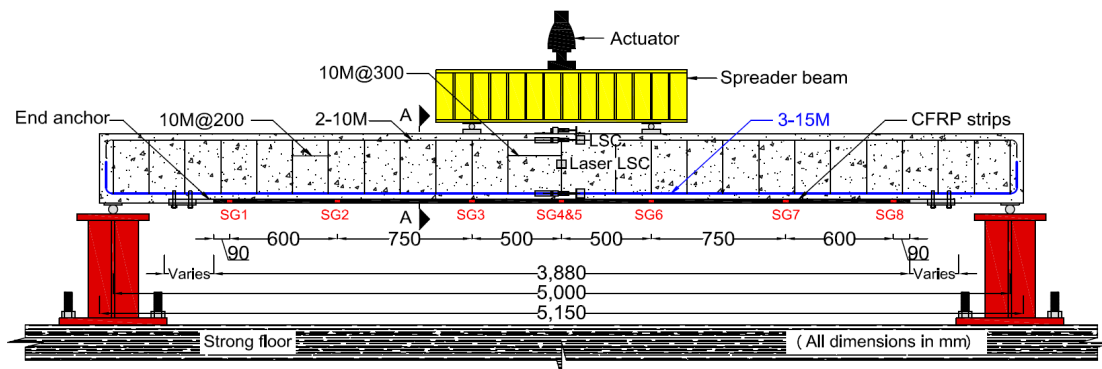


Figure 1. Test setup and geometry of the beams.

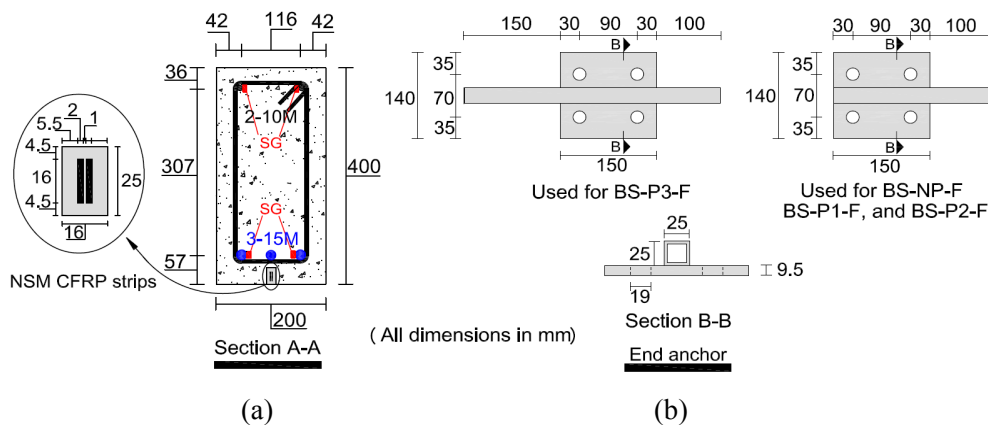


Figure 2. Cross-section of the beams and anchor details.

3 DESCRIPTION OF ALGORITHM

The developed analytical model generates the load-deflection response of the prestressed NSM-CFRP strengthened RC beams using numerical integration of the curvatures along the length of the beam. The model has the capabilities of assigning the actual concrete stress-strain curve based on Loov's equation (Loov, 1991), elasto-plastic behaviour for the compression and tension steel, linear-elastic behaviour for the FRPs, and different prestressed NSM-CFRP lengths along the length of the beam. Since the overall flexural behaviour of the tested beams is not affected by debonding (Omran & El-Hacha, 2012a), the perfect bond is assumed in the analytical model, therefore, two failure modes (CFRP rupture or concrete crushing) are considered. The model is developed for beams tested under four-point bending configuration, but it can be modified for different types of loading configuration by making few changes. The main advantage of the developed analytical model versus equivalent finite element analysis is in having less computer computational time.

The algorithm includes seven steps to produce the overall load-deflection response and the computational source code is written in Wolfram Mathematica (Wolfram Research, 2008), a powerful automated technical computing software. The inputs include twenty-five constants, which represent material properties and geometry of the beam. The code is written based on different variables, arrays for loads, deflections, moments, and curvatures, different loops and functions available in the software. The output is set to present the type of failure; a plot of the load-deflection response; a plot of the curvatures along the length of the beam at failure; and load, deflection, moment, and mid-span curvature for twenty-four points on the load-deflection curve including prestressing, cracking, yielding, and ultimate stages. A concise illustration for calculation of the load-deflection response is provided in this paper and the computational source code is not published due to limitation in the paper length.

3.1 Concepts for generating load-deflection response

The mid-span deflection of a beam at an arbitrary load level is calculated by integration of curvatures along the length as presented in Equation (1), where the curvature at every point is calculated using Equation (2). Equation (1) is a generalization of the moment-area theorem, representing the deflection of the support from the tangent to the axis of the member at mid-span, and applies whether elastic or plastic curvatures occur. To calculate the mid-span deflection of a RC beam with partial prestressing length of the NSM-CFRP along the span and under four-point bending configuration as shown in Figure 3, Equation (1) can be expanded to Equation (3):

$$\Delta = \int_0^{L/2} \phi(x) x dx \quad (\text{Park \& Paulay, 1975}) \quad (1)$$

$$\phi(x) = \frac{M(x)}{EI(x)} \quad (\text{Park \& Paulay, 1975}) \quad (2)$$

$$\Delta = - \underbrace{\int_{L_o}^{L/2} \frac{M_p(x)}{E_c I_{gt}} x dx}_I + \underbrace{\int_0^{x_{cr}} \frac{M(x)}{E_c I_{gt}} x dx}_II + \underbrace{\int_{x_{cr}}^{L_p} \frac{M(x)}{EI(x)} x dx}_III + \underbrace{\int_{L_p}^{L/2} \frac{M(L_p)}{EI(L_p)} x dx}_IV \quad (3)$$

where Δ is the deflection, x the distance from the support, $\phi(x)$ the curvature at distance x , L the span length, $M(x)$ the applied moment at distance x , $EI(x)$ the flexural stiffness at distance x , $M_p(x)$ the applied moment on the beam at distance x due to prestressing, L_o the un-strengthened length, E_c the modulus of elasticity of concrete, I_{gt} the moment of inertia of the gross

transformed section, x_{cr} the distance from the support to a point where the applied moment is equal to the cracking moment capacity of the section, L_p the distance from the support to the point load, $M(L_p)$ the moment value at point load location, and $EI(L_p)$ is the flexural stiffness at point load location (see Figure 3).

The upward deflection at mid-span due to prestressing is calculated using part I of Equation (3) in which it is assumed that the beam remains un-cracked. Integration of curvatures along the un-cracked length of the beam is calculated using part II of Equation (3). Contribution of the cracked length of the beam in the resulted deflection is computed using parts III and IV of Equation (3), in which part IV is related to the constant moment regions. In the developed analytical model, first, the cracking, yielding, and ultimate capacities of the beam are calculated. Then, the mid-span deflections are calculated at 10th point between cracking to yielding loads on the load-deflection curve, and also, at 10th point between yielding to ultimate loads. To calculate the corresponding deflection for an arbitrary applied load (e.g., at a load value between yielding and ultimate), the integration limit x_{cr} in Equation (3) needs to be specified for the applied load. This integration limit is calculated based on the moment diagram and knowing the cracking moment capacity of the section (e.g., under four-point bending $x_{cr}=M_{cr}/P_{applied}$) as shown in Figure 3. After specifying the integration limits, Equation (3) can be solved by knowing the flexural stiffness ($EI(x)$) for parts III and IV related to the cracked regions of the beam. The value of EI in the cracked region depends on the applied moment and curvature at each section, which changes from a point to another point along the cracked length ($EI(x)=M(x)/\varphi(x)$). Therefore, to solve integrals III and IV of Equation (3), the cracked length of the beam is divided to equal segments (small lengths) and assumed that the curvature is constant along each small length. Afterwards, the applied moment at the center of each small length is easily calculated by having the moment diagram of the applied load. Then, the curvature (and also EI) at the center of each small length was calculated by applying the force and moment equilibriums of the section and finding the unknowns (ϵ_c , the concrete strain at extreme compression fiber, and c , depth of the neutral axis) at each small length ($\varphi_{segment}=\epsilon_c/c$, $EI_{segment}=M_{segment}/\varphi_{segment}$). In the developed code, for each applied load, part III of Equation (3) is calculated by dividing the integral from x_{cr} to L_p to fifty integrals and part IV is calculated by one integral (since the curvature is constant along the integration limits in part IV). The number of the segments (fifty for part III) is selected based on a sensitivity analysis on the output. It should be noted that the load-deflection response is generated based on twenty four different applied loads and corresponding deflections.

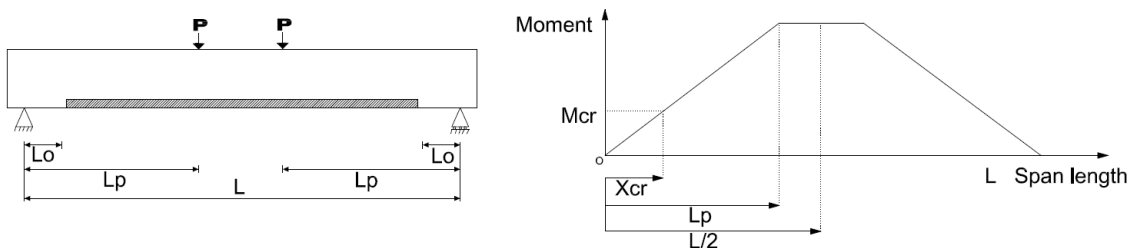


Figure 3. Finding the integration limits for Equation (3) using moment diagram.

It should be mentioned that Equation (3) is the simplified form of a more detailed equation for calculation of the mid-span deflection. In fact, in a beam that is not strengthened for the entire length with prestressed NSM-CFRP, both of the strengthened and un-strengthened portions of the length should be considered in calculation of the deflection. If the cracks form within the un-strengthened length of the beam (the applied moment along the un-strengthened length is larger than the cracking moment capacity of the un-strengthened section), therefore, parts II and III of

Equation (3) should be replaced with Equation (4). On the other hand, if no cracks form within the un-strengthened length of the beam, parts II and III of Equation (3) should be replaced with Equation (5). For the beams that properly strengthened (same as the ones employed in this paper), Equation (5) can be simplified to Equation (6) and be used in Equation (3) with insignificant effect on the resulted deflection. The latter is employed in this research.

$$II + III = \int_0^{x_{cr-un}} \frac{M(x)}{E_c I_{gt-un}} x dx + \int_{x_{cr-un}}^{L_o} \frac{M(x)}{EI(x)} x dx + \int_{L_o}^{x_{cr-st}} \frac{M(x)}{E_c I_{gt-st}} x dx + \int_{x_{cr-st}}^{L_p} \frac{M(x)}{EI(x)} x dx \quad (4)$$

$$II + III = \int_0^{L_o} \frac{M(x)}{E_c I_{gt-un}} x dx + \int_{L_o}^{x_{cr-st}} \frac{M(x)}{E_c I_{gt-st}} x dx + \int_{x_{cr-st}}^{L_p} \frac{M(x)}{EI(x)} x dx \quad (5)$$

$$II + III = \int_0^{x_{cr-st}} \frac{M(x)}{E_c I_{gt-st}} x dx + \int_{x_{cr-st}}^{L_p} \frac{M(x)}{EI(x)} x dx \quad (6)$$

where x_{cr-un} and x_{cr-st} are the distance from the support to a point where the applied moment is equal to the cracking moment capacity of the un-strengthened and strengthened sections, respectively, I_{gt-un} and I_{gt-st} the moment of inertia of the gross transformed un-strengthened and strengthened sections, respectively, and the other parameters are described earlier.

4 MODELING OF MATERIALS

4.1 Concrete of exposed beam

The exposed concrete stress-strain curve was defined based on Loov's equation. The concrete compressive strength of the exposed beams was obtained using hammer test performed on the specimens. It should be mentioned that concrete cylinders were subjected to the same environmental conditioning as the beams but their strength do not represent the concrete compressive strength of the large-scale beams. The other properties of the exposed concrete including modulus of elasticity and strain at peak stress were calculated based on the study performed by Duan et al. (2011) on the effects of freeze-thaw cycles on the stress-strain curves of unconfined and confined concrete. Since the freeze-thaw cycle used by Duan et al. (2011) was different than the one conducted in this study, therefore, the equivalent number of the cycles (N) is obtained using Equation (7) by having the concrete compressive strength at different stages. The strain at peak stress and modulus of elasticity after exposure were calculated using Equations (8) and (9). Finally by finding the properties of the exposed concrete and apply two points of the actual stress-strain curve to the Loov's equation, Equation (10) is derived for the exposed concrete. More details about the procedure for deriving the concrete stress-strain curve using Loov's equation can be found in Loov (1991) and Omran & El-Hacha (2012b). Therefore, a modulus of elasticity, ultimate strain, and tensile strength of 26.3 GPa, 0.00361, 3.79 MPa were assigned to the exposed concrete in the analytical model, respectively.

$$N = \left(1 - f_{c \text{ exposed}} / f_{c \text{ unexposed}}\right) / \left(200 f_{c28}^{-3.0355}\right) \quad (\text{Duan et al., 2011}) \quad (7)$$

$$\epsilon_{0 \text{ exposed}} = \epsilon_{0 \text{ unexposed}} e^{(661.742 f_{c28}^{-5.1406} N)} \quad (\text{Duan et al., 2011}) \quad (8)$$

$$E_{c \text{ exposed}} = E_{c \text{ unexposed}} e^{(-1.1345 \times 10^7 f_{c28}^{-5.7089} N)} \quad (\text{Duan et al., 2011}) \quad (9)$$

$$f_c = 40 (709.86 \epsilon_c) / \left(1 + 173.22 \epsilon_c + 2.08 \times 10^{12} \epsilon_c^{4.91}\right) \quad (10)$$

where N is the number of freeze-thaw cycles, $f_{c \text{ exposed}}$ the concrete compressive strength after exposure (MPa), $f_{c \text{ unexposed}}$ the concrete compressive strength before exposure (MPa), f_{c28} the concrete compressive strength at 28 days (MPa), $\epsilon_{0 \text{ exposed}}$ the concrete strain at peak stress after exposure, $\epsilon_{0 \text{ unexposed}}$ the concrete strain at peak stress before exposure, $E_{c \text{ exposed}}$ the modulus of elasticity of concrete after exposure (MPa), $E_{c \text{ unexposed}}$ the modulus of elasticity of concrete before exposure (MPa), f_c the concrete compressive stress (MPa), and ϵ_c is the strain at any concrete compressive stress f_c .

4.2 CFRP strip

A linear elastic behaviour was considered for the CFRP strips with ultimate strain, modulus of elasticity, and total area of 0.021, 124.4 GPa, and 62.4 mm², respectively (Omran & El-Hacha, 2012a).

4.3 Steel reinforcement

An elasto-plastic behaviour was considered for the steel reinforcements in the analytical model with yield strain, modulus of elasticity, and total area of 0.00244, 200 GPa, and 200 mm² for the compression steel, and 0.00246, 200 GPa, and 600 mm² for the tension steel, respectively (Omran & El-Hacha, 2012a).

5 NONLINEAR ANALYSIS

The nonlinear analysis was performed by satisfying the moment and the force equilibriums for cross-section of the beam shown in Figure 4 and finding the unknowns (concrete strain at extreme compression fiber and depth of the neutral axis). These equilibrium equations are presented in Equations (11) and (12). The components of Equations (11) and (12) are calculated using the Equations (13)-(18).

$$\sum F = 0 \Rightarrow T_f + T_s - C_s - C_c = 0 \quad (11)$$

$$\sum M = 0 \Rightarrow C_c \bar{y}_{Cc} + C_s (c - d_{sc}) + T_f (d_f - c) + T_s (d_{st} - c) = M \quad (12)$$

$$T_f = A_{fip} E_{fip} (\epsilon_f + \epsilon_{pe}) \quad (13)$$

$$T_s = A_{st} E_{st} \epsilon_{st} \quad \text{if } \epsilon_{st} < \epsilon_{yt} \quad \text{or} \quad A_{st} f_{yt} \quad \text{if } \epsilon_{st} \geq \epsilon_{yt} \quad (14)$$

$$C_s = A_{sc} E_{sc} \epsilon_{sc} \quad \text{if } \epsilon_{sc} < \epsilon_{yc} \quad \text{or} \quad A_{sc} f_{yc} \quad \text{if } \epsilon_{sc} \geq \epsilon_{yc} \quad (15)$$

$$C_c = \int_0^c b f_c(y) dy \quad (16)$$

$$\bar{y}_{Cc} = \left(\int_0^c y b f_c(y) dy \right) / \left(\int_0^c b f_c(y) dy \right) \quad (17)$$

$$f_c(y) = 40 \left(709.86 \left(\frac{\epsilon_{cc} y}{c} \right) \right) / \left(1 + 173.22 \left(\frac{\epsilon_{cc} y}{c} \right) + 2.08 \times 10^{12} \left(\frac{\epsilon_{cc} y}{c} \right)^{4.91} \right) \quad (18)$$

where T_f is the force in CFRP strips, T_s the force in tension steel bars, C_s the force in compression steel bars, C_c the compressive force carried by concrete, $\bar{y}_{Cc}$ the distance between neutral axis and point of action of the resultant compressive force on concrete, c the depth of neutral axis, d_{sc} the depth to the centroid of the compression steel bars, d_f the depth to the

centroid of the CFRP strips, d_{st} the depth to the centroid of the tension steel bars, M the applied moment; A_{frp} , E_{frp} , ϵ_f , and ϵ_{pe} the area (mm^2), the modulus of elasticity (MPa), the strain, and the prestrain of the CFRP strips, respectively; A_{st} , E_{st} , ϵ_{st} , f_{yt} , and ϵ_{yt} the area (mm^2), the modulus of elasticity (MPa), the strain, the yield stress (MPa), and the yield strain of the tension steel bars, respectively; A_{sc} , E_{sc} , ϵ_{sc} , f_{yc} , and ϵ_{yc} the area (mm^2), the modulus of elasticity (MPa), the strain, the yield stress (MPa), and the yield strain of the compression steel bars, respectively; b the width of the beam (mm), $f_c(y)$ the concrete compressive stress at height y defined based on Equation (18) (MPa), ϵ_{cc} the concrete strain at extreme compression fiber, y the height from the neutral axis (mm), and c is the depth of the neutral axis (mm).

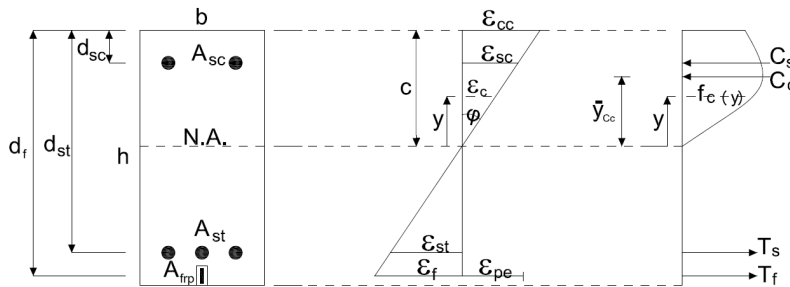


Figure 4. Strain and stress distribution on a prestressed NSM-CFRP strengthened section.

6 RESULTS AND DISCUSSION

Comparison between experimental and analytical load-deflection responses is presented in Figure 5. The analytical solutions of the beams were terminated due to concrete crushing or CFRP rupture whichever occurred first. The estimated load-deflection responses include the negative camber due to prestressing, initiation of flexural cracks, yielding of tension steel bars, and failure at ultimate stage.

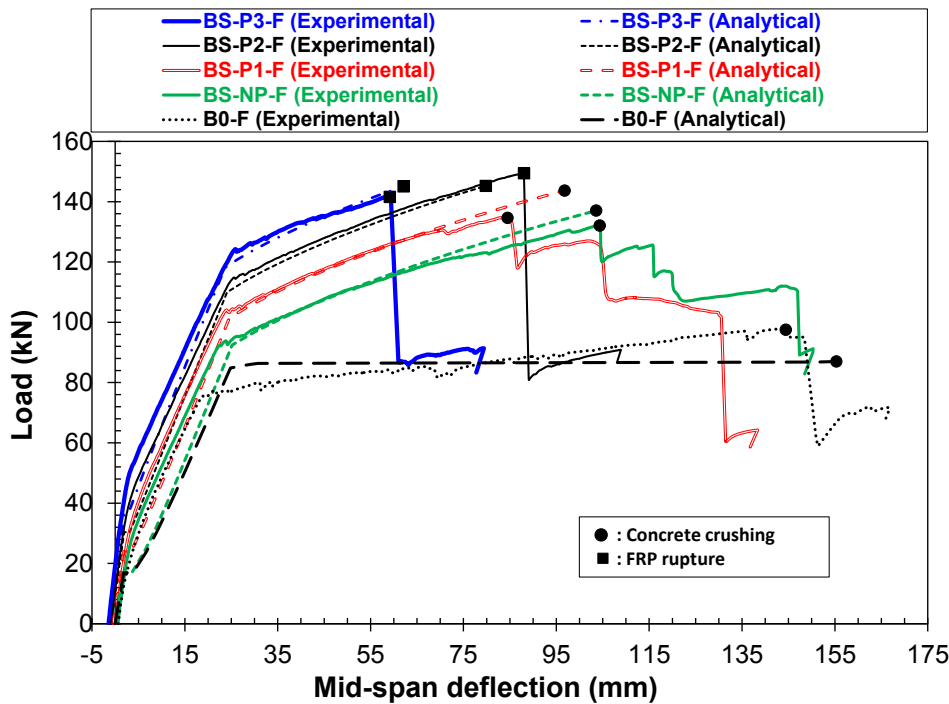


Figure 5. Comparison between experimental and analytical load-deflection responses.

A summary of the results obtained from the tests versus the analytical solutions are presented in Table 1 including type of the failure, ductility index (the ratio of the ultimate deflection to the deflection at yielding), energy absorption (the area under load-deflection curve up to the peak load), and the percentage of difference between corresponding experimental and analytical values.

Table 1. Summary of the results

Beam ID#	ϵ_{peff} (10^6)	Results	Δ_{peff} (mm)	P_{cr} (kN)	Δ_{cr} (mm)	P_y (kN)	Δ_y (mm)	P_u (kN)	Δ_u (mm)	μ	ψ (kN. mm)	FM
B0-F	N.A.	Analytical	0	16.9	1.81	75.5	18.78	97.8	142.88	7.61	11646.7	CC
		Test*	0	10.4	2.22	84.8	24.81	87.0	155.31	6.26	12368.6	CC
		Error %	0	62.4	-18.5	-10.9	-24.3	12.4	-8.0	21.6	-5.8	
BS-NP-F	0	Analytical	0	17.1	1.81	92.4	25.19	137.1	103.59	4.11	10398.9	CC
		Test*	0	14.0	1.36	92.4	22.53	132.2	104.26	4.63	10649.0	CC
		Error %	0	22.0	33.4	0.0	11.8	3.7	-0.6	-11.1	-2.3	
BS-P1-F	3463	Analytical	-0.48	23.9	1.87	101.5	24.61	143.7	96.77	3.93	10419.3	CC
		Test*	-0.49	21.6	1.43	104.1	23.99	134.7	82.90	3.46	8667.1	CC
		Error %	-1.7	10.6	31.2	-2.5	2.6	6.7	16.7	13.8	20.2	
BS-P2-F	6723	Analytical	-0.94	30.4	1.92	109.9	24.09	145.2	79.81	3.31	8774.7	FR
		Test*	-1.09	27.3	1.24	114.8	25.56	149.5	87.85	3.44	10214.1	FR
		Error %	-14.2	11.2	54.9	-4.3	-5.7	-2.9	-9.2	-3.6	-14.1	
BS-P3-F	9884	Analytical	-1.38	36.6	1.97	118.1	23.62	145.1	62.11	2.63	6832.1	FR
		Test*	-1.70	35.5	1.45	124.3	25.91	141.7	58.55	2.26	6509.8	FR
		Error %	-19.1	3.1	35.9	-5.0	-8.8	2.4	6.1	16.4	5.0	

ϵ_{peff} and Δ_{peff} = effective prestrain and camber at one week after prestressing, P_{cr} and Δ_{cr} = load and deflection at cracking, P_y and Δ_y = load and deflection at yielding, P_u and Δ_u = load and deflection at ultimate, μ = ductility index (Δ_u / Δ_y), ψ = energy absorption (area under P- Δ curve up to P_u), FM = failure mode, CC = concrete crushing, FR=CFRP rupture

*(Omran & El-Hacha, 2012a)

At cracking, a relatively large percentage of difference is observed between experimental and analytical values. Considering the strengthened beams, an average error of $11.7 \pm 7.8\%$ for cracking load with a maximum of 22% for BS-NP-F, and an average error of $38.8 \pm 10.9\%$ for cracking deflection with a maximum of 54.9% for BS-P2-F are observed. Also, differences of 62.4% and -18.5% are observed for cracking load and deflection of B0-F, respectively. The high percentage of the difference at cracking stage might be due to presence of the micro cracks in the beams before testing. The other reason for underestimation or overestimation of the cracking load using the analytical solution might be due to difference between concrete compressive strength in the model and in tested beams. The beams were cracked after strengthening prior to being subjected to freeze-thaw exposure, while in the analytical solution an average exposed concrete compressive strength was assigned to the beams (40 MPa for all beams), that might be slightly different with reality, at the time of initial cracking for each beam. The resulted difference between analytical solution and test values at cracking is most possibly the accumulation of the above mentioned reasons. At yielding stage, the differences between analytical solutions and experimental results are negligible. Considering the strengthened beams, an average error of $-3 \pm 2.2\%$ for yield load with a maximum of -5% for BS-P3-F and an average error of $0 \pm 9.2\%$ for yield deflection with a maximum of 11.8% for BS-NP-F are reached.

At ultimate stage, the predicted loads are almost the same as those from the test; however, the predicted ultimate deflections are different than those from the test values. Since the failure governs by CFRP rupture or concrete crushing, the CFRP ultimate strain value and the concrete-

stress-strain curve have a significant impact on the predicted ultimate deflections. Therefore, a slight difference between the assigned material properties (i.e., CFRP ultimate tensile strain or concrete compressive stress-strain curve) with reality for each beam would lead to a difference at ultimate deflection. For instance, as presented in Omran & El-Hacha (2012a), beams BS-P2-F and BS-P3-F did not experimentally fail exactly at the CFRP ultimate strain of 0.021, which is assigned to the analytical model. Considering the strengthened beams, an average error of $2.5 \pm 4\%$ for ultimate load with a maximum of 6.7% for BS-P1-F, and an average error of $3.3 \pm 10.9\%$ for ultimate deflection with a maximum of 16.7% for BS-P1-F are observed at the ultimate stage. The modeled beams showed similar type of failure to the tested beams. Furthermore, average errors of $7.4 \pm 13.4\%$ and $0.6 \pm 12.9\%$ for ductility index (μ) and energy absorption (ψ) are reached considering all beams. The fluctuation of the experimental curve at ultimate stage is not observed in the analytical solution, which is mainly due to elimination of probable local debonding in experiment and assuming complete bond in analytical model. Therefore, performed comparison indicates that the load-deflection curves obtained from the analytical solutions can accurately predict those from the experimental ones.

7 CONCLUSIONS

A nonlinear analytical model was developed to generate the load-deflection responses of RC beams strengthened in flexure with prestressed or non-prestressed NSM-CFRP strips subjected to freeze-thaw exposure. The model has the capabilities of assigning the freeze-thaw exposed concrete stress-strain curve based on Loov's equation, elasto-plastic behaviour for compression and tension steel, linear behaviour for FRP, and partial prestressed CFRP length along the length of the beam. Also, it has main advantage of having much shorter computer computational time in comparison with finite element analysis of identical beams. The reliability of the model was confirmed by comparing five experimental load-deflection responses with the analytical results revealing very good accuracy of the predicted results.

The finding of this paper reveals that the proposed analytical model can be employed with enough confidence as a predictive method in future studies.

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