Second Conference on Smart Monitoring,
Assessment and Rehabilitation of Civil Structures

SMAR 2013

Nonlinear analysis of semi-rigid frames with rigid end sections accounting for shear deformations

H Görgün[1], S Çelik[2], A Çelik[3]

[1] Department of Civil Engineering, Dicle University, 21280, Diyarbakir, Turkey
[2] Department of Civil Engineering, Dicle University, 21280, Diyarbakir, Turkey
[3] Department of Civil Engineering, Dicle University, 21280, Diyarbakir, Turkey

ABSTRACT: Moment-rotation behaviour of beam-to-column connections used in precast concrete and/or steel structures plays an important role in the analysis and design of these structures as this is the most important influence on the response of either individual members or complete plane frames. A computer-based method is presented for geometrically nonlinear plane frames with semi-rigid connections accounting for shear deformations and rigid end sections. The material nonlinearity of the members and connections are considered in the analysis. The analytical procedure employs modified stability functions to model the effect of axial force on the stiffness of members. The member stiffness matrix is evaluated in local coordinate system then transferred to the global coordinate system using the transformation matrix. Finally the global system stiffness matrix is assembled. The fixed end forces for various loads commonly used in practice were found. The geometrically and materially nonlinear analysis method is applied for one planar structure. The method is readily implemented on a computer using matrix structural analysis techniques and is applicable for the efficient nonlinear analysis of frameworks.

1 GENERAL INSTRUCTIONS

In the current design and analysis of structural systems the members forming the precast concrete and/or steel planar frames are general1y assumed to be rigidly connected among each other. However, more often than not the assumption of pin connections is also employed in such cases where the rigidity of the connection cannot be provided to a dependable degree. In fact, both of the foregoing assumptions are unrealistic when one is treating steel frames and especially, nowadays, widely used precast reinforced concrete structures. In such structures beams and columns behave as if they are semi-rigidly, or flexibly, connected among themselves, as far as rotations of the ends are concerned. Hence, experimentally determined effective rotational spring constants for those connections should be used in the analyses of such structures. The present study is an attempt to prepare a computer program that treats the aforementioned type of structures elegantly, taking into consideration the behaviour of the flexible connections and the effect of shear deformations along with the effect of geometric nonlinearity due to the axial forces in the members, the material and beam-to-column connection nonlinearities.

The method used in the present study is the well-known stiffness method of structural analysis. First, the stiffness matrix of a bar elastically supported against rotation at both ends is obtained using the second order analysis. Then, the fixed end forces are found for a bar elastically supported at the two ends by rotational springs for various loads. For the latter analysis, the second order theory was employed once again, along with the use of differential equations which yielded trigonometric functions for the case of compressive axial force and hyperbolic functions for the case of tensile axial force.

The computer program that was prepared can be used to solve static problems of planar frames composed of members that are flexibly connected at the nodes.

2 PREVIOUS STUDIES

Cunningham (1990) has carried out some experiments on flexibly connected steel frames. From this study graphical presentation of moment-relative rotation relations have been obtained for different kinds of connections among steel members.

A different kind of study, though relevant for the present one, was carried out by Dincer (1989), in which the author considered planar frames with members having rigid end sections (which resemble parts of lintel beams that extend from the face of a shear wall to its axis) taking into consideration the geometric nonlinearity in the case of combined flexural and shear deformations.

Although the study mentioned above does not involve flexible connections, it has given inspiration to the present study by the methods applied, namely the techniques of applying differential equations to the elastic parts of the members, separately for compressive and tensile axial forces, and the iterative treatment of the nonlinearity in the computer program.

Görgün, Yilmaz and Karacan (2012) studied the nonlinear analysis of frames with rigid end sections and nonlinear semi-rigid connections taking into consideration the geometric nonlinearity and shear deformations, and implemented their analysis by a computer program.

Görgün and Yilmaz (2012) studied the nonlinear analysis of planar frames composed of flexibly connected members taking into consideration the geometric nonlinearity and shear deformations, and implemented their analysis by a computer program.

Görgün (2013) studied the nonlinear analysis of planar frames composed of flexibly connected members taking into consideration the geometric nonlinearity and implemented his analysis by a computer program. The author founds the stiffness matrix for a single bar with rotational springs at the ends, using the pertinent differential equations. Then, he founds the fixed end forces for a concentrated load, a uniformly distributed load, a linearly distributed load, a symmetrical trapezoidal distributed load and a nonsymmetrical triangular distributed load.

3 METHOD OF APPROACH

The present study is mainly composed of two parts. The first part is comprised of the analytical study that employs the matrix method which is commonly used in structural analysis. In this part, the stiffness matrix of the structure of concern is obtained, the contributions of different types of loads to the loading vector are found and the formulation of the equilibrium equations for the determination of the unknown displacements is explained. In this part of the study the axial forces in the members causing the nonlinearity do increase the difficulty of the computations a great deal. Actually, besides the more complicated type of functions compared to linear analysis, there is also a need for separate analyses for compressive and tensile axial forces which doubles the analytical work. In the second part of the study the pertinent computer program was prepared.

4 RESEARCH FINDINGS, DISCUSSION AND RESULTS

4.1 Analysis

In the present study, the method used being the stiffness method, the main concern is to set up the relation between the loading and the displacement vectors of a given structure. To accomplish this, the first thing to be done is to find the relation between the end forces and the end displacements for a member. Towards this end we must first define the sign convention and notation which is done in Figure 1. As is well known, the end forces p of a straight member in terms of the end displacements d and fixed end forces f, due to intermediate loads, is given by the well known formula,

$$p = kd + f \tag{1}$$

where k is a six by six matrix whereas p, d and f are six by one vectors. Letting $y = y_m + y_s$ show the downwards displacements, the deflection due to bending only is shown by y_m and that due to shear is shown by y_s, and x show the distance from the left end of the member, one can find the different elements of the stiffness matrix by taking each and every end displacement to be unity at a time, when the others are zero and solving the differential equation

$$y'' = y''_m + y''_s = -\frac{M}{EI\left(1 - P/GA_s\right)} \tag{2}$$

where a prime shows a derivative with respect to x and EI is the flexural rigidity of the member.

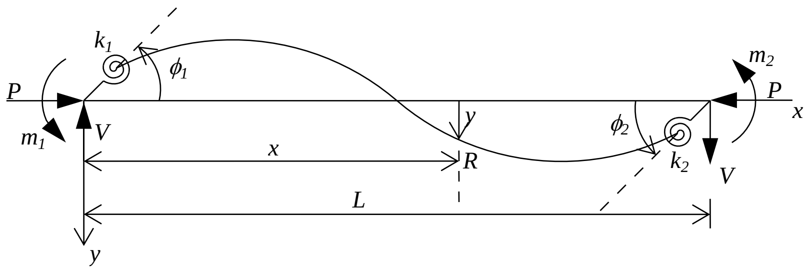

Figure 1: Notation for a member with axial force.

When there is axial force, the expression for bending moment for any cross-section along the member, like the one at point R, is (see Figure 1)

$$M = \pm Py + Vx - m_1 \tag{3}$$

where P is the absolute value of the axial force and the sign in front of it in Equation (3) is positive for compression and negative for tension. Defining

$$\alpha = \begin{cases} \sqrt{\dfrac{P/EI}{\left(1 - P/GA_s\right)}} & P < 0 \\ \sqrt{\dfrac{P/EI}{\left(1 + P/GA_s\right)}} & P > 0 \end{cases} \tag{4}$$

the general solution of Equation (2) is

$$y = A\sin\left(\alpha x\right) + B\cos\left(\alpha x\right) - \frac{V}{P}x + \frac{m_1}{P} \tag{5}$$

when the axial force is compressive.

When the axial force is tensile and the last term in the bending moment expression in Equation (3) changes sign, then the general solution of Equation (2) is again given by Equation (5) only changing the signs of the last two terms and the trigonometric functions to their corresponding hyperbolic ones. Assigning the unit end displacements to the outer ends of the springs, each at a time and using the equilibrium equations for the free body diagrams of the members along with Equation (5) and the suitable boundary conditions for the displacements and slopes at the inner ends of the springs, the stiffness matrices are found in the same form for the two cases, and defining

$$\begin{aligned}
\chi_1 &= \psi^3\delta^2\left\{\left(1-\psi^2\beta_1\beta_2\right)\sin\psi+\psi\left(\beta_1+\beta_2\right)\cos\psi\right\}\\
\chi_2 &= \psi^2\delta\left(\psi\beta_2\sin\psi-\cos\psi+1\right)\\
\chi_3 &= \psi^2\delta\left(\psi\beta_1\sin\psi-\cos\psi+1\right)\\
\chi_4 &= \psi\left\{\left(1+\psi^2\delta\beta_2\right)\sin\psi-\psi\delta\cos\psi\right\}\\
\chi_5 &= \psi\left(\psi\delta-\sin\psi\right)\\
\chi_6 &= \psi\left\{\left(1+\psi^2\delta\beta_1\right)\sin\psi-\psi\delta\cos\psi\right\}\\
\Omega &= \psi\left\{\delta\left(\psi^2\beta_1\beta_2-1\right)+\beta_1+\beta_2\right\}\sin\psi-\left\{2+\psi^2\delta\left(\beta_1+\beta_2\right)\right\}\cos\psi+2
\end{aligned} \tag{6}$$

for the case of compressive axial force and

$$\begin{aligned}
\chi_1 &= \psi^3\delta^2\left\{\left(1+\psi^2\beta_1\beta_2\right)\sinh\psi+\psi\left(\beta_1+\beta_2\right)\cosh\psi\right\}\\
\chi_2 &= \psi^2\delta\left(\psi\beta_2\sinh\psi+\cosh\psi-1\right)\\
\chi_3 &= \psi^2\delta\left(\psi\beta_1\sinh\psi+\cosh\psi-1\right)\\
\chi_4 &= -\psi\left\{\left(1-\psi^2\delta\beta_2\right)\sinh\psi-\psi\delta\cosh\psi\right\}\\
\chi_5 &= -\psi\left(\psi\delta-\sinh\psi\right)\\
\chi_6 &= -\psi\left\{\left(1-\psi^2\delta\beta_1\right)\sinh\psi-\psi\delta\cosh\psi\right\}\\
\Omega &= \psi\left\{\delta\left(\psi^2\beta_1\beta_2+1\right)-\beta_1-\beta_2\right\}\sinh\psi-\left\{2-\psi^2\delta\left(\beta_1+\beta_2\right)\right\}\cosh\psi+2
\end{aligned} \tag{7}$$

for the case of tensile axial force, the modified local–axis stiffness matrix for the member is,

$$k=\begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{EI\chi_1}{L^3\Omega} & \frac{EI\chi_2}{L^2\Omega} & 0 & -\frac{EI\chi_1}{L^3\Omega} & \frac{EI\chi_3}{L^2\Omega} \\ 0 & \frac{EI\chi_2}{L^2\Omega} & \frac{EI\chi_4}{L\Omega} & 0 & -\frac{EI\chi_2}{L^2\Omega} & \frac{EI\chi_5}{L\Omega} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{EI\chi_1}{L^3\Omega} & -\frac{EI\chi_2}{L^2\Omega} & 0 & \frac{EI\chi_1}{L^3\Omega} & -\frac{EI\chi_3}{L^2\Omega} \\ 0 & \frac{EI\chi_3}{L^2\Omega} & \frac{EI\chi_5}{L\Omega} & 0 & -\frac{EI\chi_3}{L^2\Omega} & \frac{EI\chi_6}{L\Omega} \end{bmatrix} \tag{8}$$

where

δ : $1-\mathrm{P}/\mathrm{GA_s}$ for compressive and $1+\mathrm{P}/\mathrm{GA_s}$ for tensile axial force

$\psi=\alpha L$, $\beta_1=\frac{1}{4k_1}$, $\beta_2=\frac{1}{4k_2}$

k : Cross-sectional constant of the member

4.2 Modified stiffness matrix of a member with rigid end sections

Shear walls are usually connected by beams and for the purposes of analysis we have to find the stiffness of such a beam corresponding to coordinates at the wall axis.

The rotations and the translations, parallel to the axis of the member, at the ends of either rigid section are equal to each other. Hence, it can easily be proved that the stiffness matrix of member *i*j** is also symmetrical and its elements which are different from those of member *ij* can be found as follows:

$$\begin{aligned} k_{23}^* &= k_{23}+k_{22}(dL)=k_{32}^*=-k_{35}^*=-k_{53}^* \\ k_{26}^* &= k_{26}+k_{22}(bL)=k_{62}^*=-k_{56}^*=-k_{65}^* \\ k_{33}^* &= k_{33}+2k_{23}(dL)+k_{22}(dL)^2\pm P(dL) \\ k_{36}^* &= k_{36}+k_{23}(bL)+k_{26}(dL)+k_{22}(dL)(bL)=k_{63}^* \\ k_{66}^* &= k_{66}+2k_{26}(bL)+k_{55}(bL)^2\pm P(bL) \end{aligned} \tag{9}$$

where P is the absolute value of the axial force in the member and the sign in front of it in Eq. (12) is positive for tension, negative for compression, and vanishes ($P=0$) for linear solution.

Concerning fixed end forces for numerous types of span loadings, although the computations involved are rather tedious, the method of approach is straightforward and simple. What needs to be done in each case is to employ the method used for finding the stiffness matrix, namely apply Equation (2) where bending moment M. given by Equation (3), is expressed with an additional term or terms due to the span loading and the force P at the left end is found by using the moment equilibrium equation relative to the right end. The respective boundary conditions for all cases are zero vertical displacements and rotations which are proportional to the end moments having the correct sign, at both ends. For cases necessitating two regions with different differential equations the additional conditions at their common point are the equivalence of their deflections and slopes. Moreover, for the case of symmetrical trapezoidal distributed load, in making use of symmetry, the midspan slope was taken to be zero. The fixed end moments, can be found in the M.S. thesis of the second author. The corresponding transverse forces can be found by making use of the two equations of equilibrium for the member.

4.3 Programming

The analytical expressions having been prepared for all the quantities of relevance for the problem, it remained only to write down a computer program for numerical applications. That was done and the resulting program contains special differences compared to a linear analysis. The main difference is that there is an iteration which can be stopped when a desired accuracy is reached. The geometric stiffness matrix, as it is called, due to axial force is a relevant feature of this analysis, which actually is the cause of the necessity for the iterative procedure. The analysis starts with zero axial forces in all members giving the linear solution at this first step. Then, at each new step the axial forces found in the previous step are used in the computations, of both the stiffness matrix and the fixed end forces. When a predetermined precision is attained, the final node displacements, member end forces, and variations of bending moment along relevant members are determined. The maximum value of the bending moment in each member is given, along with the maximum value and its position on the member. The source program is available from either of the authors. It cannot be presented here due to its excessive length.

4.4 Numerical results and discussion

The linear and nonlinear analysis procedures are illustrated in the following example structure comprised of lintel beams having fully rigid end sections connected to wide column members (shear walls) with rigid and semi-rigid connections. The example is a shear wall with opening, a six-story single-bay building framework, the linear and nonlinear analysis of which have been extensively studied in the literature from a variety of different computational viewpoints for which analytical results found using the computer programme are compared with other analytical results, Girijavallabhan (1969); Popov et al. (1979); Dincer (1989). The geometry and the loading of the system being given in Figure 2(a). the coding of the system is seen in Figure 2(b).

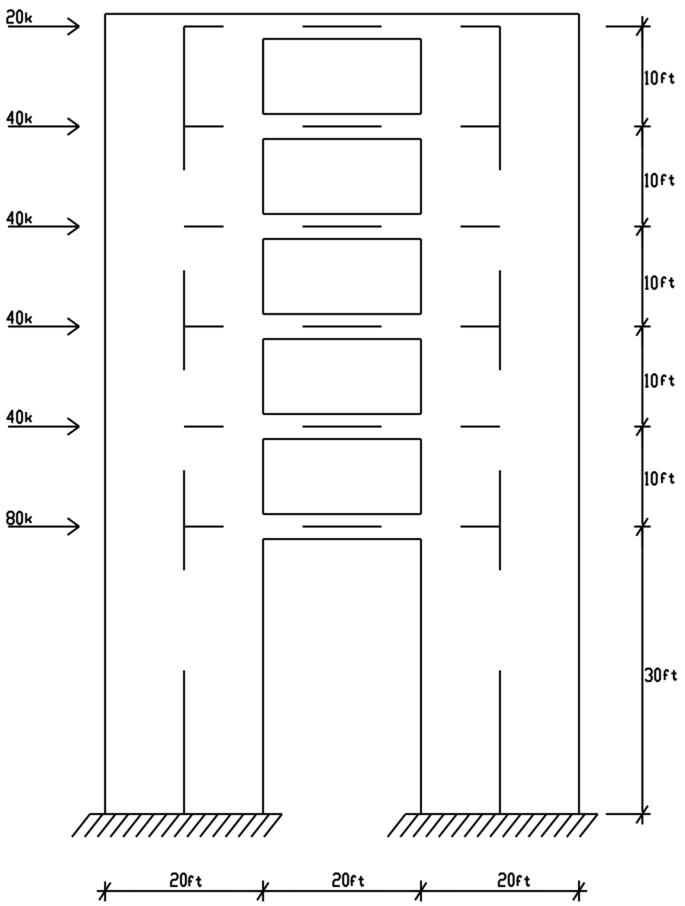

Figure 2(a): Geometry and loading of the example problem.

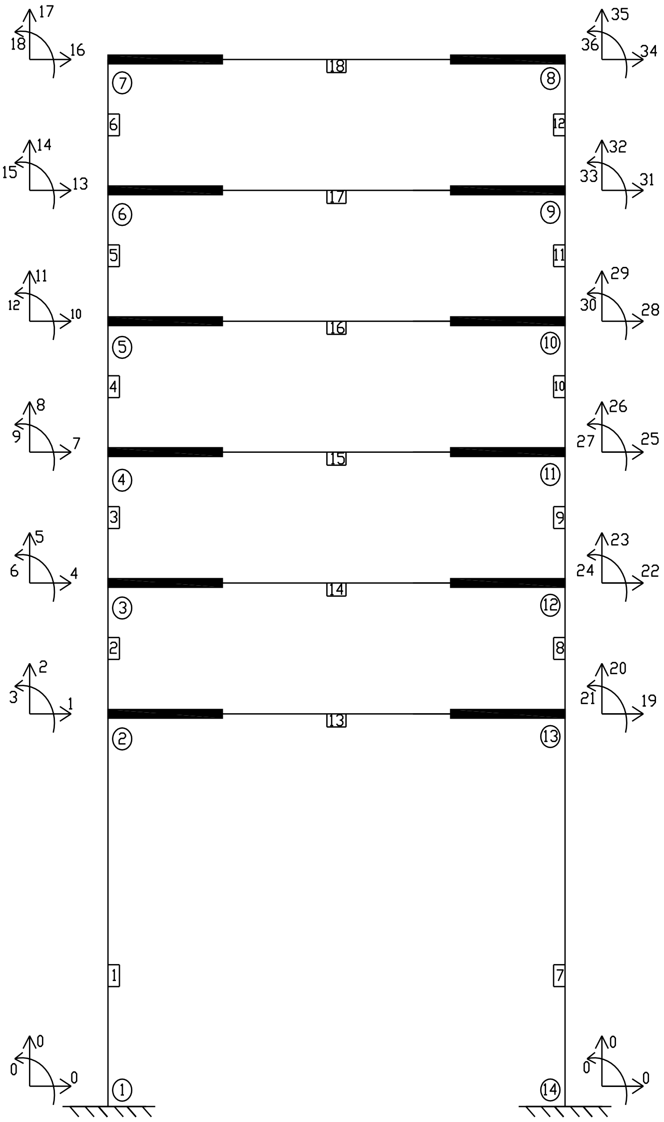

Figure 2(b): Coding and numbering of the example problem.

All lintel beams and wide columns have rectangular shape sections that are oriented in the plane of the framework and are assumed to be fully restrained against out-of-plane behaviour with the following properties: lintel beams section depth $h = 2$ ft, the thickness of the wall was assumed to be equal to 1.0 ft., section area $A = 2\ \text{ft}^2$, moment of inertia $I = 0.667\ \text{ft}^4$, and shape factor $f = 5/6$. Wide columns section area $A = 20\ \text{ft}^2$, moment of inertia $I = 667.667\ \text{ft}^4$. Poisson's ratio $\nu = 0.15$. The framework has 18 members, 14 nodes and 36 degrees-of-freedom (dof) for nodal displacement (i.e. lateral and vertical translation and rotation dof at each of the twelve free nodes 2-13). Briefly discussed in the following are the results of the study that demonstrate analytically the influence that shear, semi-rigid connections, and the geometrically nonlinear have on the behaviour of the member end forces. The number of iterations for improving the axial forces is chosen to be 5.

The resulting end forces for the members are presented in Tables 1-2. Other results like nodal displacements and span moments cannot be given due to restriction on the number of pages.

Table 1. Comparison of member end moments with rigid connections for linear frame analysis.

Lintel beam	Considering the effect of shear deformation ($\nu = 0.15$)					
	Grijavallabhan (1969)		Popov et al. (1979)		Dincer (1989) and Present Study	
	m_1 (kip-ft)	m_2 (kip-ft)	m_1 (kip-ft)	m_2 (kip-ft)	m_1 (kip-ft)	m_2 (kip-ft)
13	38.78	38.48	62.70	62.50	62.00	61.76
14	41.10	40.89	69.32	69.25	69.29	69.25
15	41.33	41.06	72.26	72.35	72.83	72.90
16	40.29	39.79	72.90	73.05	73.80	73.94
17	38.75	38.27	72.17	72.37	73.37	73.59
18	31.37	31.60	61.59	62.91	72.73	72.97

Table 2. Comparison of member end moments with rigid connections for linear and nonlinear frame analyses.

Lintel beam	Member end moments (kip-ft)							
	Linear $\nu = 0$		Linear $\nu = 0.15$		Nonlinear $\nu = 0$		Nonlinear $\nu = 0.15$	
	m_1	m_2	m_1	m_2	m_1	m_2	m_1	m_2
13	63.44	63.06	62.00	61.76	59.96	59.73	57.66	57.53
14	70.80	70.70	69.29	69.25	66.21	66.15	64.51	64.48
15	74.34	74.39	72.83	72.90	69.60	69.63	68.21	68.24
16	75.26	75.43	73.80	73.94	71.05	71.15	69.72	69.81
17	74.79	75.03	73.37	73.59	71.53	71.69	70.16	70.30
18	74.10	74.38	72.73	72.97	72.01	72.22	70.74	70.92

To give an idea about the effect of spring constants, on the displacements, the variations of the lateral displacements of six nodes of the structure with varying spring constants for all springs in the structure have been presented in Figure 3.

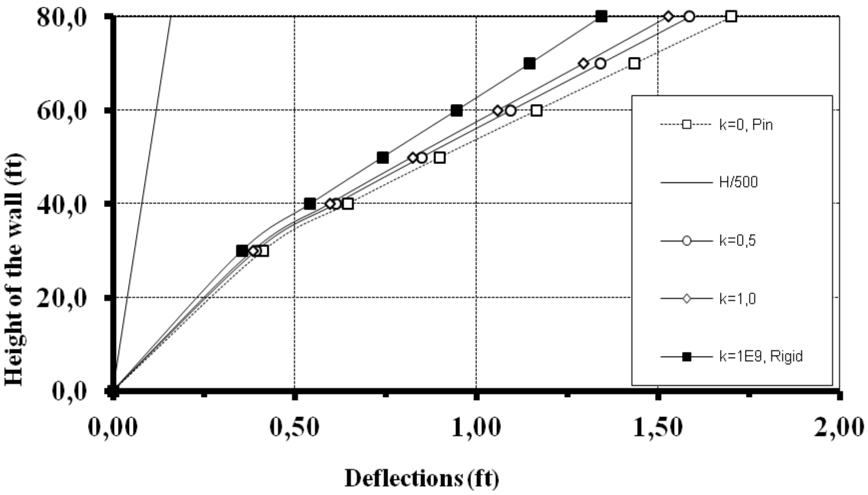

Figure 3: Sway displacements at each floor level in the example problem with varying spring constants k

4.5 Conclusions

Analysis by conventional simple bending theories might lead to an overestimation or underestimation of the end forces in shear walls and lintel beams. In this study the second order analysis of planar frames made up of flexibly connected prismatic members having rigid end sections taking into consideration the effect of shear deformations and material nonlinearity is considered and a computer program is prepared for numerical computations. Different types of span loadings are considered and most of the span loadings not being found in the literature, the results are checked among themselves as special cases of others. Moreover, special problems being mirror images of others are used for checking purposes, as well. A design example is included to demonstrate effect of connection flexibility, rigid end sections, shear deformations, the geometrical nonlinearity and the material nonlinearity in the design of general frames.

It is noticed from the design example that semi-rigid connection flexibility affects the distribution of forces in the frame and causes increase in the drift of the frame. This in turn necessitates the consideration of effect in the frame analysis. It required three to five iterations in the design examples considered to obtain the nonlinear response of frame which clearly indicates the significance of geometric nonlinearity in the analysis and design of semi-rigid frames. Comparisons with results found by other methods for the frame example determined that the proposed method can effectively predict the member end forces of general frameworks, achieve more accurate results than the conventional method.

References

Cunningham, R. 1990. Some aspects of semi-rigid connections in structural steelwork. The Structural Engineer, 68(5):85-92.

Dincer, R. 1989. Nonlinear analysis of planar frames with linear prismatic members having rigid end sections taking shear deformation into consideration. MSc Thesis, University of Cukurova, Adana, Turkey.

Girijavallabhan, CV. 1969. Analysis of shear wall with openings. J Struct Div ASCE, 95(10): 2093-2103.

Gorgun, H, Yilmaz, S, Karacan, SS. 2012. Nonlinear analysis of frames composed of flexibly connected members with rigid end sections accounting for shear deformations, Scientific Research and Essays, 7(27), 2390-2408.

Gorgun, H, and Yilmaz, S. 2012. Geometrically nonlinear analysis of plane frames with semi-rigid connections accounting for shear deformations, Structural Engineering and Mechanics, Vol. 44, No. 4, 539-569.

Gorgun, H. 2013. Geometrically nonlinear analysis of plane frames composed of flexibly connected members, Structural Engineering and Mechanics, Vol. 45, No. 3, 273-305.

Popov, EP, Petersson H, and Le DQ. 1979. Program Subwall, Finite element analysis of structural walls. ACI Journal, 76(30): 679-696.