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Integrated Analysis of Inhomogeneous Structural Monitoring Data from Internal and External Sensors

Werner Lienhart

Graz University of Technology, Graz, Austria

ABSTRACT: In modern structural monitoring geodetic sensors (total stations, GNSS sensors, etc.) are used simultaneously with other sensors like level sensors or fiber optic sensors. Traditionally, the deformation analysis is based on network adjustment and focuses on the geometrical changes of an object neglecting the causes for the deformations and the dynamics of the process. However, for a detailed understanding of the structural behavior the data from all sensors as well as a numerical model of the structure have to be incorporated in the analysis. In this paper an integrated analysis method (IAM) is presented which allows the use of different types of data in combination with a finite element model (FEM).

1 INTRODUCTION

Traditionally, geodetic monitoring of structures is based on point measurements on the surface of an object. Therefore, prisms or GNSS antennas are mounted on the structure. The 3D movements of these points can be observed continuously or epoch-wise with robotic total stations or GNSS receivers. Alternatively the surface of the object can be monitored without attaching targets with laser scanners or by photogrammetric techniques. Standard method in geodetic monitoring is to establish a control network where all network points (stable reference points and moveable object points) are connected (Cooper, 1987). A typical setup for the geodetic monitoring of a civil structure is shown in fig. 1.

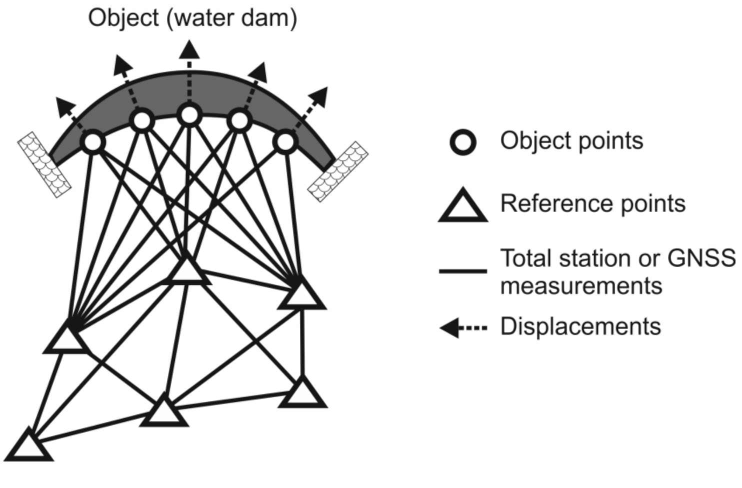

Figure 1. Geodetic monitoring network for a water dam (after Welsch et al., 2000, p. 65).

In recent years attempts were made to extend the geodetic measurement domain by including sensors that can also be embedded within structures. An example are fiber optic sensors (FOS) which can be placed in many civil structures like concrete bridges or earth filled dams. These sensors enable deformation measurements in areas which are not accessible by conventional geodetic techniques. To date we have experiences in the combined use of geodetic and fiber optic sensors in bridge monitoring (Lienhart and Brunner, 2003), landslide monitoring (Wöllner et al., 2011) and dam monitoring (Lienhart, 2013). In this paper we demonstrate that standard analysis techniques fail to analyze the combined data in an integrated manner and propose an integrated analysis method (IAM) which allows the use of different types of data in combination with a finite element model (FEM).

2 CONVENTIONAL GEODETIC DEFORMATION MONITORING

Geodetic measurements are usually carried out in an over-determined manner, e.g. monitoring points are observed from two or more reference stations. Thus the least squares method can be applied to estimate the best result in terms of minimum sum of squares of the residuals. The well-known solution (see e.g. Niemeier, 2001) to this minimization problem is

$$\hat{\boldsymbol{\xi}} = (\boldsymbol{A}^{\mathrm{T}} \boldsymbol{\Sigma}_{\boldsymbol{yy}}^{-1} \boldsymbol{A})^{-1} \boldsymbol{A}^{T} \boldsymbol{\Sigma}_{\boldsymbol{yy}}^{-1} \boldsymbol{y} = \boldsymbol{N}^{-1} \boldsymbol{A}^{T} \boldsymbol{\Sigma}_{\boldsymbol{yy}}^{-1} \boldsymbol{y} \tag{1}$$

with

$\hat{\boldsymbol{\xi}}$ estimated parameters

$\boldsymbol{y}$ observations

$\boldsymbol{\Sigma}_{yy}$ variance co-variance matrix of the observations ($\boldsymbol{\Sigma}_{\boldsymbol{yy}} = \sigma_0^2 \boldsymbol{V}_{yy}$)

$\boldsymbol{A}$ design matrix

$\boldsymbol{N}$ normal equation matrix

The design matrix $\boldsymbol{A}$ is the crucial element which establishes the connection between observations and parameters. It contains the partial derivatives of the observations with respect to the parameters:

The estimated parameters (typically coordinates) and their variance co-variance matrix (VCM) are the core elements of the deformation analysis. If the geometry of an object is measured at two points in time (i and j) then deformation can be defined as difference between the estimated parameters of the individual measurement epochs:

$$\boldsymbol{d}_{ij} = \hat{\boldsymbol{\xi}}_j - \hat{\boldsymbol{\xi}}_i \tag{2}$$

In the congruency model (Pelzer, 1971) it is verified by a statistical test if the calculated deformations are significant. If the null hypothesis of no significant deformation is rejected moved object points are separated from stable points. Fig. 2 summarizes this standard procedure.

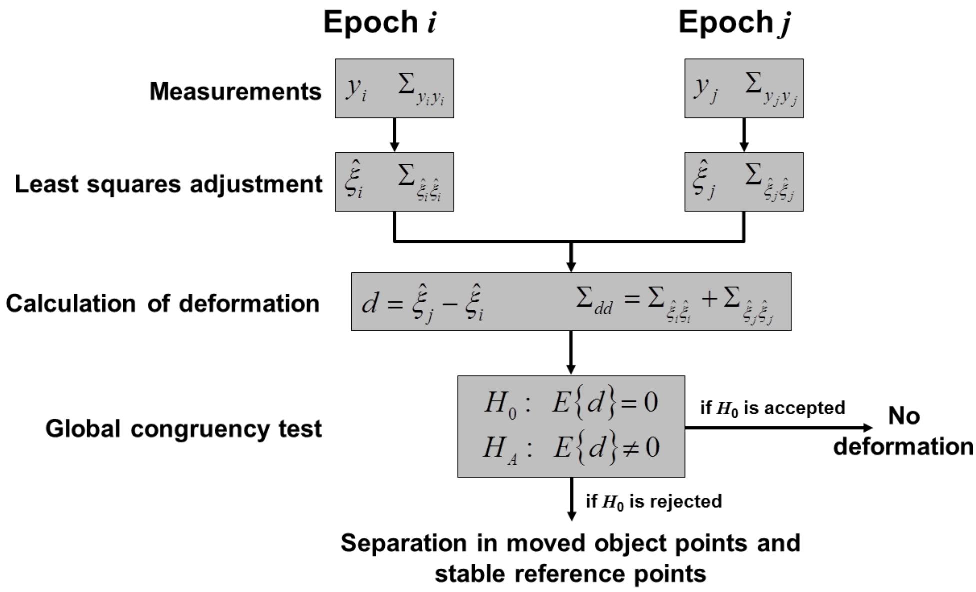

Figure 2. Deformation analysis with the congruency model.

The congruency model is a proven tool for the analysis of conventional geodetic monitoring data where all individual points are part of the geodetic network. However, this method fails in case of external and internal measurements. In many applications it is not possible to perform connection measurements between internal and external sensors. Furthermore, connection measurements between different internal sensors are also difficult to achieve. Such a situation is visualized in fig. 3. In this case two points P_1 and P_2 on the object are monitored with total station measurements (direction (Hz) and distance (D) measurements). Additional strain measurements (ε) are taken with the internal sensors L_1 and L_2.

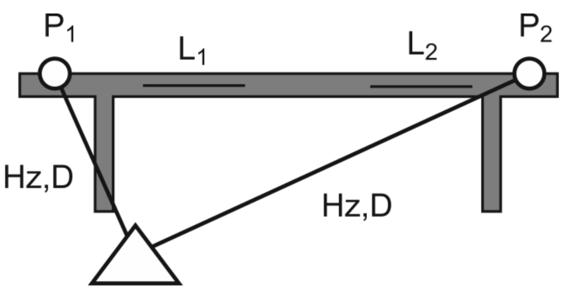

Figure 3. Monitoring example with internal and external measurements.

The linearized model for the least squares adjustment

$$\boldsymbol{y} + \boldsymbol{e} = \boldsymbol{A} \cdot \boldsymbol{\xi} \tag{3}$$

to determine the point coordinates $\boldsymbol{x}$ and the length of the internal sensors l_1 and l_2 is given by

$$\begin{bmatrix} \boldsymbol{y}_{Hz} \\ \boldsymbol{y}_{D} \\ \boldsymbol{y}_{\varepsilon_1} \\ \boldsymbol{y}_{\varepsilon_2} \end{bmatrix} + \boldsymbol{e} = \begin{bmatrix} \boldsymbol{A}_{Hz} & & \\ \boldsymbol{A}_{D} & & \\ & \boldsymbol{A}_{\varepsilon_1} & \\ & & \boldsymbol{A}_{\varepsilon_2} \end{bmatrix} \cdot \begin{bmatrix} \hat{\boldsymbol{x}} \\ \hat{\boldsymbol{l}}_1 \\ \hat{\boldsymbol{l}}_2 \end{bmatrix} \tag{4}$$

The normal equation system collapses due to the block structure of the design matrix

$$\boldsymbol{N} = \boldsymbol{A}^T \cdot \boldsymbol{V}_{yy}^{-1} \cdot \boldsymbol{A} = \begin{bmatrix} \boldsymbol{N}_{xx} & & \\ & \boldsymbol{N}_{l_1 l_1} & \\ & & \boldsymbol{N}_{l_2 l_2} \end{bmatrix} \tag{5}$$

As a result the data is not analyzed in an integrated manner. In fact the different parameter types can be calculated independently due to the diagonal structure of the global normal equation matrix by solving the following equations

$$\begin{aligned} \hat{\boldsymbol{x}} &= \boldsymbol{N}_{xx}^{-1} \left(\boldsymbol{A}_{Hz}^T \cdot \boldsymbol{V}_{HzHz}^{-1} \cdot \boldsymbol{y}_{Hz} + \boldsymbol{A}_D^T \cdot \boldsymbol{V}_{DD}^{-1} \cdot \boldsymbol{y}_D \right) \\ \hat{\boldsymbol{l}}_1 &= \boldsymbol{N}_{l_1 l_1}^{-1} \boldsymbol{A}_{\varepsilon_1}^T \cdot \boldsymbol{V}_{\varepsilon_1 \varepsilon_1}^{-1} \cdot \boldsymbol{y}_{\varepsilon_1} \\ \hat{\boldsymbol{l}}_2 &= \boldsymbol{N}_{l_2 l_2}^{-1} \boldsymbol{A}_{\varepsilon_2}^T \cdot \boldsymbol{V}_{\varepsilon_2 \varepsilon_2}^{-1} \cdot \boldsymbol{y}_{\varepsilon_2} \end{aligned} \tag{6}$$

The information that all measurements are performed on the same object is lost in the standard deformation analysis method.

3 INTEGRATED ANALYSIS

For a better understanding of structural behavior the observed object has to be treated as dynamic system which converts acting forces into deformations. These deformations can be predicted with a numerical model if the acting forces $\boldsymbol{f}_{MEAS}$ are measured and compared with actually measured deformations. One possibility of a numerical model with physically meaningful parameters is a finite element model (FEM) of the structure. An FEM calculation delivers nodal displacements $\boldsymbol{u}$ from which calculated deformations $\boldsymbol{d}_{FEM}$ can be derived. In general the calculated deformations $\boldsymbol{d}_{FEM}$ will differ from the measured deformations $\boldsymbol{d}_{MEAS}$. The significance of this difference $\boldsymbol{\delta}$ has to be verified by a statistical test, see fig. 4. In order to perform this test, the variance co-variance matrix $\boldsymbol{\Sigma}_{\delta\delta}$ has to be known. This matrix is the sum of the VCM of the measured deformations and the VCM of the predicted deformations. The VCM of the predicted deformations has to be derived from the VCM of the acting forces $\boldsymbol{\Sigma}_{ff}$ by variance propagation through the model and does also include uncertainties of the FEM.

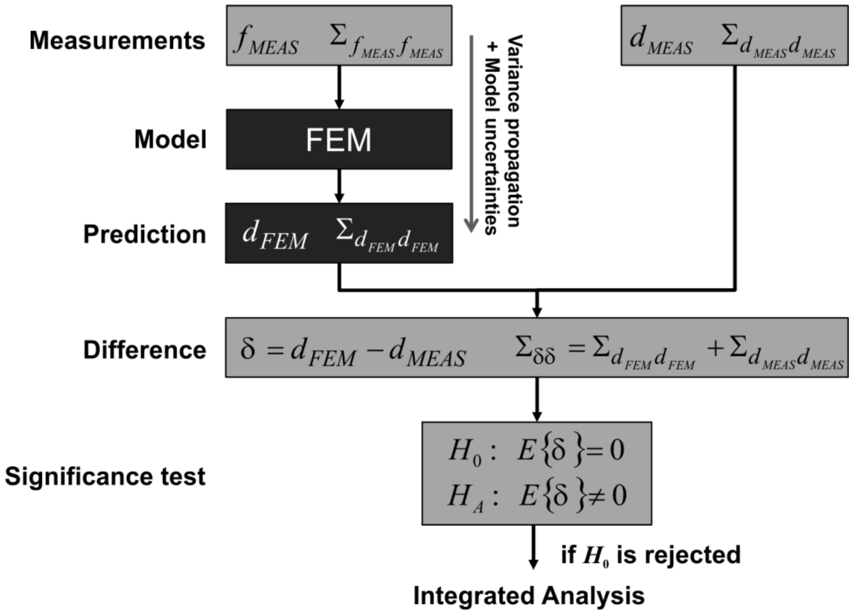

Figure 4. Comparison of calculated and measured deformations.

In case of significant differences an Integrated Analysis has to be performed. The Integrated Analysis can be based on adaptive Kalman-filtering (Heunecke, 1995) if continuous measurements are available or can be based on the Integrated Analysis Method (IAM) presented by Lienhart (2007) in case of a small number of measurement epochs. The basic idea in an IAM is to introduce the condition that the calculated deformations have to be identical with the measured deformations when the calibrated parameters of the numerical model are used. Brandes et al. (2012) formulate this condition explicitly and introduce Lagrangian multipliers. This condition can also be realized by pseudo-observations with assigned high weight (Jäger and Bertges, 2004).

This method differentiates between a measurement and a system part and also uses the variance co-variance matrices (VCM) of the measurements. The measurement part consists of the measured deformations $\boldsymbol{d}_{MEAS}$ which should be equivalent to the deformations calculated from the nodal displacements $\boldsymbol{u}$. Generally, the unknown parameters of the FEM will cause a difference which can be used to estimate the parameter values $\hat{\boldsymbol{p}}$. A Gauss Markov model can be introduced which uses the equivalence of the sum of the measured deformations $\boldsymbol{d}_{MEAS}$ and the unknown residuals $\boldsymbol{e}_{MEAS}$ to the deformations calculated from estimated nodal displacements $\hat{\boldsymbol{u}}$ (eq. 7).

Measurement part

$$\boldsymbol{d}_{MEAS} + \boldsymbol{e}_{MEAS} = \boldsymbol{d}_{FEM}(\hat{\boldsymbol{u}}), \qquad \boldsymbol{\Sigma}_{dd,MEAS} \tag{7}$$

System part

$$\boldsymbol{u}_{SYS} + \boldsymbol{e}_{SYS} = \hat{\boldsymbol{u}} - \boldsymbol{K}(\hat{\boldsymbol{p}})^{-1} \cdot \hat{\boldsymbol{f}}, \quad \boldsymbol{\Sigma}_{SYS} \tag{8}$$

$$\boldsymbol{p} + \boldsymbol{e}_p = \hat{\boldsymbol{p}}, \qquad \boldsymbol{\Sigma}_{pp} \tag{9}$$

$$\boldsymbol{f} + \boldsymbol{e}_f = \hat{\boldsymbol{f}}\,, \qquad\qquad \boldsymbol{\Sigma}_{ff} \tag{10}$$

Eq. 8 forces the estimated nodal displacements $\hat{\boldsymbol{u}}$ to be identical with the calculated nodal displacements using the estimated physical parameters $\hat{\boldsymbol{p}}$ and the estimated forces $\hat{\boldsymbol{f}}$. This equation is introduced as condition by assigning a large weight to it, meaning

$$\boldsymbol{\Sigma}_{SYS} \rightarrow \boldsymbol{0} \tag{11}$$

In the system part the material parameters (***p***) and forces (***f***) are included as pseudo observations (eq. 9 and 10). The parameters in the model are the nodal displacements ***u***, the material parameters ***p*** and the acting forces ***f***. These parameters are estimated by least squares adjustment. Measurement and system part can by identified in the design matrix which has the following structure

$$\mathbf{A} = \begin{bmatrix} \text{Measurement part} \\ \text{System part} \end{bmatrix} = \begin{bmatrix} \overset{\text{a)}}{\dfrac{\partial \mathbf{d}}{\partial \mathbf{u}}} & & \\ \mathbf{I} & -\dfrac{\partial \mathbf{u}}{\partial \mathbf{p}} & -\dfrac{\partial \mathbf{u}}{\partial \mathbf{f}} \;\text{b)} \\ & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix} = \begin{bmatrix} \overset{\text{a)}}{\mathbf{T}_{du}} & & \\ \mathbf{I} & -\mathbf{T}_{up} & -\mathbf{T}_{uf} \;\text{b)} \\ & \mathbf{I} & \\ & & \mathbf{I} \end{bmatrix} \tag{12}$$

where ***I*** is the identity matrix. The connection between the measurements and the calculated displacements is established by block a) of eq. 12. This block contains the partial derivatives $\boldsymbol{T}_{\mathrm{du}}$ of the deformations with respect to the nodal displacements. The connection between the displacements, the material parameters and the forces is established by block b) which contains the partial derivatives of the nodal displacements with respect to the parameters ($\boldsymbol{T}_{\mathrm{up}}$) and the forces ($\boldsymbol{T}_{\mathrm{uf}}$). The individual points of the FEM are linked together by the off diagonal elements of ***A***. In case of monitoring data from external and internal sensors functional relations are established between previously unconnected sensors.

4 EXAMPLE: MONITORING OF A MONOLITHIC BRIDGE

The above presented IAM was applied to the monitoring data of a monolithic bridge. This bridge was monitored with 27 geodetic points, 2 borehole inclinometer tubes, 8 fiber optic sensors. Additionally the internal bridge temperature was measured with 12 temperature sensors.

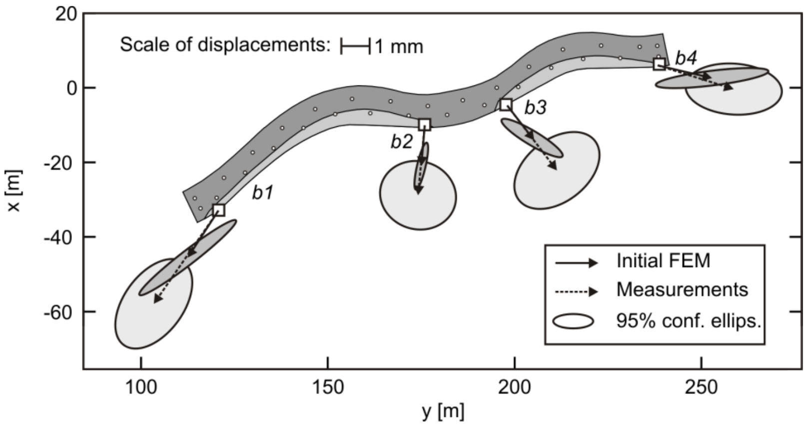

Figure 5. Calculated (initial FEM) and measured displacements.

Fig. 5 displays the deformations of the bridge deck as response to the temperature differences between autumn and summer. The deformation calculated with the initial FEM show large differences to the measured deformations. These differences are significantly reduced after the IAM due to the calibration of the FEM parameters, see fig. 6. The full analysis of this data can be found in Lienhart (2007).

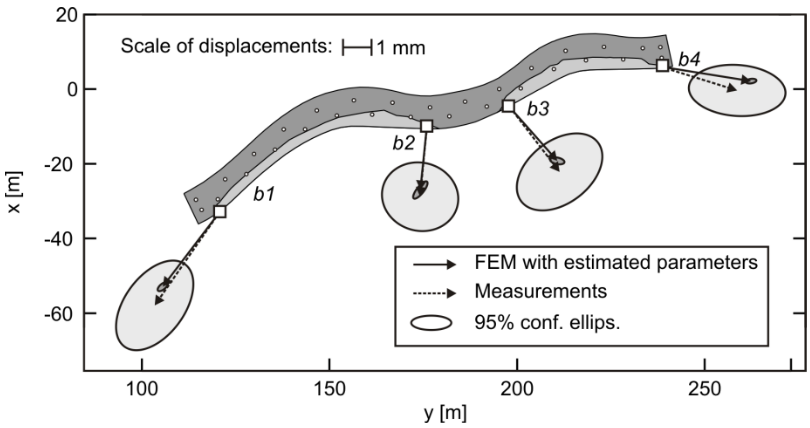

Figure 6. Calculated (calibrated FEM) and measured displacements.

5 CONCLUSIONS

Conventional deformation analysis methods fail in case of internal and external monitoring data due to a collapse of the normal equation system. The data of each sensor type is analyzed independently and thus the global deformation behavior of the structure cannot be assessed. With the proposed IAM it is possible to connect spatially distributed measurements of different types. As an example the analysis of the monitoring of a monolithic bridge was discussed in this paper. The results of the calibrated FEM are in good agreement with the measured deformations. The presented method is generic and can be applied to inhomogeneous measurements of any structure.

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