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Structural damage estimation using dynamic data and multi-objective optimization

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ABSTRACT: A novel damage estimation method is proposed using a multi-objective optimization technique which simultaneously updates the damaged as well as undamaged structural model. Contemporary damage detection and estimation methods based on model updating typically require a prior updated baseline finite element model of the undamaged structure for subsequent updating of the damaged model. However, multi-objective optimization algorithms utilize both the undamaged and damaged structure models concurrently to improve the performance of the damage estimation procedure. An updated solution is selected which gives a trade-off between updating of both models using the concept of Pareto front. The technique was applied to a numerically simulated structure with damage, and natural frequencies and mode shape information was used for model updating. Different noise levels were added to account for the experimental errors. It was found that the proposed method gives more accurate damage location and estimation than traditional updating, and is less sensitive to experimental errors.

1 INTRODUTION

For the design and analysis of civil engineering systems, the finite element (FE) method is widely used. FE models of civil structures are usually based on idealized drawings/designs and estimates of material properties, structural geometry and boundary and connectivity conditions, which may not truly reflect the exact behavior of the as-built system. Significant differences in dynamic behavior of FE models and as-built systems have been noted by various researchers (Schulne et al. 2009, Brownjohn et al. 2001, Friswell & Mottershead 1995) and numerous studies focusing on damage detection and estimation have been carried out using vibration data (Perera et al. 2009, Hu et al. 2001, Huh et al. 2011, Hester & Gonzales 2012). These differences can be mainly attributed to simplification of a complex structure and uncertainties associated with assumptions of materials, geometry, and boundary and connectivity conditions (Moon & Aktan 2006). Dynamic model updating is a process of refining the mathematical model of an actual structure using dynamic measurements.

This paper presents a novel damage estimation method which concurrently utilizes the experimental data of both undamaged and damaged structure in a multi-objective optimization (MOO) process. Contemporary damage detection and estimation methods which are based on model updating typically require an updated baseline FE model of the undamaged structure. The updated undamaged model is then compared with an updated damaged model for assessment of damage location and severity. There might be many errors associated with this two-stage approach, e.g. modeling errors, experimental errors, updating procedure errors or parametric errors. These errors may be aggregated in the subsequent model updating runs.

To check the performance of the proposed approach, it is applied to a numerically simulated simply supported beam. Different noise levels are added to the identified mode shapes to assess the performance of the proposed procedure for accurate damage estimation in noisy conditions. A genetic algorithm (GA) has been used as the optimization tool in this paper as multi-objective GA is well implemented (Knowles et al. 2008). Compared to the single-objective optimization (SOO), which gives only one optimal solution, multi-objective formulation gives a set of alternative solutions. A desirable solution can then be selected based on acceptable trade-off between, in our case, the two objective functions related to the undamaged and damaged structure, respectively.

The layout of the paper is as follows. MOO concepts are explained first. This is followed by model updating of a simulated beam using SOO and MOO under different noise levels. The results of damage estimation using SOO and MOO are then compared and conclusions drawn from the study are reported.

2 MULTI-OBJECTIVE OPTIMISATION

An optimal solution for a physical system modeled using one objective function can be found using SOO. However, when two or more objective functions are used concurrently, the task of finding one or more optimal solutions is referred to as MOO. Many real world optimization problems involve multiple objectives. The extremist principle which prioritizes one objective over the others may lead to erroneous results especially in cases where the rest of objectives are also important or interdependent. Selecting a solution which is optimal for only one objective may compromise the other objectives. Evolutionary algorithms are a popular approach to solve MOO problems using the concept of domination (Deb 2001). According to this concept, one solution dominates the other solution if the following two conditions are satisfied:

1. The first solution is not worse than the second solution in all objectives, and
2. The first solution is strictly better than the second solution in at least one objective.

Violation of any of the above conditions indicates that the first solution does not dominate the second solution. It is intuitive that if any solution dominates the other solution, then it is also better in the context of MOO. The concept of domination is used to find 'non-dominated' solutions. When all pair-wise comparisons have been made for a given finite set of solutions, we expect to have a set comprising a number of solutions which do not dominate one another. An important property of this set is that each of its solutions dominates all other solutions outside of this set. In other words, the solutions in this set are better compared to the rest of the solutions. This leads to the definition of Pareto optimality which states that among all solutions, a non-dominated set of solution are those which are not dominated by the rest of the solutions. This concept of Pareto optimality leads to a set of solutions known as the Pareto optimal set. A rank is assigned to all solutions within the set (Deb et al. 2002). A plot of objective function values corresponding to the Pareto optimal set gives the Pareto front.

Evolutionary algorithms, such as GA, work concurrently on a population of genes and use genetic operators such as selection, crossover and mutation to obtain globally optimal solutions. This evolution mechanism helps to explore the trade-off between solutions with different blends and grades of objectives. Also, they do not require gradients of the objective function and their chance to reach the global optimal solution is increased. A detailed review of multi-objective techniques can be found in Coello & Lamont (2004). Many variants of MOO, based on the Pareto front approach and using multi-objective evolutionary algorithms (MOEAs), have been proposed. Non-dominated sorted genetic algorithm-II (NSGA-II) is one of the most popular and efficient MOEAs, and has been used in many studies in the last decade (Deb et al. 2002, Chan & Sudhoff 2010, Koppen & Yoshida 2007, Hui & Qingfu 2009). Therefore, NSGA-II has been adopted for this study to investigate its effectiveness for damage detection and estimation via dynamic model updating. The general steps involved in NSGA-II are as follows:

1. Termination criteria based on the accuracy required and total number of generations are selected
2. A random population of chromosomes (solutions) is initialized
3. Values of objective functions for each of the chromosomes are obtained
4. Different ranks are assigned to each of the solutions based on a non-dominated sorting algorithm to classify the population into fronts
5. Off springs of the parent population are created by randomly arranging a duplicate copy of the parent solutions
6. A tournament selection of best solutions obtained from the previous step is performed
7. Cross over with assumed probability is performed on the parent solutions to form new off springs
8. The new off springs are mutated with a mutation probability
9. A non-dominated sorting is performed on the new off springs and once again all the solutions are classified into fronts using a non-dominated sorting algorithm
10. If the termination criteria are achieved, stop, or else go to step 6.

After some trial and error, the following parameters of NSGA-II have been used in the present research:

Population size = 500

Minimum value of objective function = 1×10^{-10}

Maximum number of generations = 200

Cross over probability = 0.8

Mutation probability = linearly decreases from 0.2 to 0 when the maximum number of generations is reached

Pareto fraction (fraction of solutions to be kept in the first front) = 0.35.

3 DAMAGE ESTIMATION USING MULTI-OBJECTIVE OPTIMIZATION

In the previous decade, numerous studies have been performed to establish the health of the structure under in-situ conditions (Hu et al. 2001, Begambre & Laier 2009). In the context of dynamic FE model updating, assessment of physical characteristics of the structure is done by comparing basic modal properties (such as natural frequencies and mode shapes) with their FE model counterparts. In a traditional sequential model updating approach, individual model updating of the initial FE model is carried out using modal data related to undamaged structure. Likewise, the updated healthy structure FE model is again updated by considering modal data related to the damaged structure. A comparison is then made between the two individually and sequentially updated FE models to estimate damage. Different errors may propagate during individual model updating runs, such as modeling errors, experimental errors, updating procedure errors or parametric errors. In the proposed approach, the initial FE model of the structure is updated simultaneously by considering the modal data related to both undamaged and damaged structure in a multi-objective context. Two consecutively updated models thus obtained (undamaged and damaged FE model) are then compared for damage severity estimation.

A numerically simulated simply supported beam has been studied to demonstrate the effectiveness of the damage estimation method. The simulated beam has a total length of 5 m and was discretized into 10 elements as shown in Figure 1. The beam has a total depth of 0.2 m and width of 0.25 m. The density of the beam was assumed as 2500 kg/m^3 and modulus of elasticity as 3.2×10^4 MPa. The area of the cross section was 0.05 m^2 and moment of inertia (MOI) was 1.66×10^{-4} m^4. A preliminary model of the beam was assumed as the one which has the aforementioned section properties and is referred to as the initial FE model (Figure 1a). In the simulated 'experimental' model for the undamaged structure, the MOI of all the elements of the initial FE model has been reduced by 10% as shown in Figure 1b. A 10% reduction was assumed in the undamaged model as initial FE models are usually developed based on design drawings and assumptions and may not accurately represent the undamaged in-situ structure. For the simulated 'experimental' damaged structure, MOI of element No. 5 has been further reduced from 10% to 60% (Figure 1c). This methodology is advantageous to check the effectiveness of the proposed approach in updating both undamaged and damaged models simultaneously. Modal analysis was carried out on the undamaged and damaged beam to obtain the first five natural frequencies and mode shapes. The values of the first five natural frequencies for the initial FE model, undamaged model and damaged model are shown in Table 1. It can be seen that the difference between all the frequencies of the initial FE model and the undamaged beam is 5.4%, and for the damaged beam it varies between 6.8% and 17.5%. Only vertical degrees of freedoms are considered in this study as these are the only typically measured in actual tests.

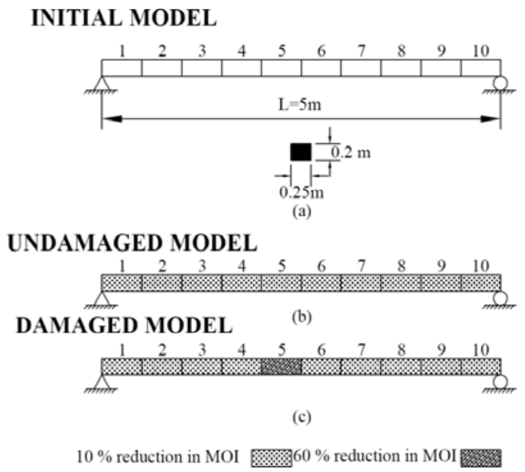


Figure 1. Simulated simply supported beam: (a) initial FE model, (b) undamaged model with 10% reduction in MOI of all elements, and (c) damaged model with 60% reduction in MOI of element No. 5 and 10% in all other elements.

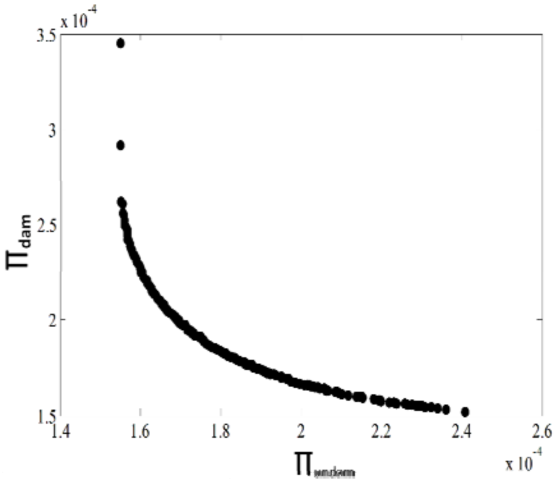


Figure 2. Typical Pareto front between two objective functions related to undamaged and damaged beam.

Table 1. Frequencies of the simulated beam before model updating.

Mode No.	Frequencies of initial FE model (Hz)	Frequencies of undamaged beam (Hz)	Frequencies of damaged beam (Hz)	Difference in frequencies between initial FE and undamaged model (%)	Difference in frequencies between initial FE and damaged model (%)
1	12.98	12.31	11.04	-5.41	-17.51
2	51.91	49.24	48.60	-5.41	-6.81
3	116.73	110.74	103.13	-5.41	-13.19
4	207.14	196.51	189.42	-5.41	-9.36
5	322.11	305.58	291.86	-5.41	-10.37

It can be assumed that modal frequencies are fairly accurately determined in modal testing and experimental errors are usually present only in mode shapes (Udwadia 2005). Consequently, some random noise has been added to each of the k-th component of the j-th modal amplitude and the ‘measured’ components of the mode shapes are given as:

$$\Phi_{jk,noise} = \Phi_{jk}\left(1+\alpha_{noise}\varepsilon\right) \tag{1}$$

where ε is a random number between -1 and +1 and α_{noise} is the amount of noise. Two different noise levels were considered, i.e. 5% and 20% (Perera & Torres 2006), for checking the effectiveness of the proposed approach. According to the usual procedure performed in actual tests, five data sets were considered for each noise level representing what would be repeated experiments. The updating parameter values reported later are the average of the five sets and their standard deviations.

A combined objective function related to the frequencies and model assurance criterion (MAC) (Möller & Friberg 1998) is used in this study. The relative error between the experimental and analytical frequencies is:

$$\Pi_1 = \sum_{i=1}^{n}\left[\left(\omega_{a,i}-\omega_{e,i}\right)/\omega_{e,i}\right]^2 \tag{2}$$

where ω represents modal frequency, subscripts a and e refer to the analytical and experimental values, respectively, and n is the total number of modes to be updated. The second objective function is related to the difference in mode shapes and can be defined in terms of MAC as:

$$\Pi_2 = \sum_{i=1}^{n}\left(1-\sqrt{MAC_i}\right)^2 \Big/ MAC_i \tag{3}$$

Two separate objective functions were defined for undamaged Π_{undam} and damaged structure Π_{dam} as follows:

$$\Pi_{undam} = \alpha\Pi_{1,undam} + \beta\Pi_{2,undam} \tag{4}$$

$$\Pi_{dam} = \alpha\Pi_{1,dam} + \beta\Pi_{2,dam} \tag{5}$$

where weighting factors α and β were both taken as 1.

Model updating of the undamaged and damaged beam is performed in this section. The first five frequencies and mode shapes were selected to have a similar number of unknowns (MOI for each FE) as the number of knowns (frequencies and MACs). This was chosen because the performance of many minimum searching algorithms deteriorates when the number of unknowns to the number of unknowns increases. Damage detection and localization capabilities of the algorithm were checked by including all the parameters related to undamaged and damaged beam in the process. Both SOO and MOO were performed to obtain the updated parameters to compare their performance.

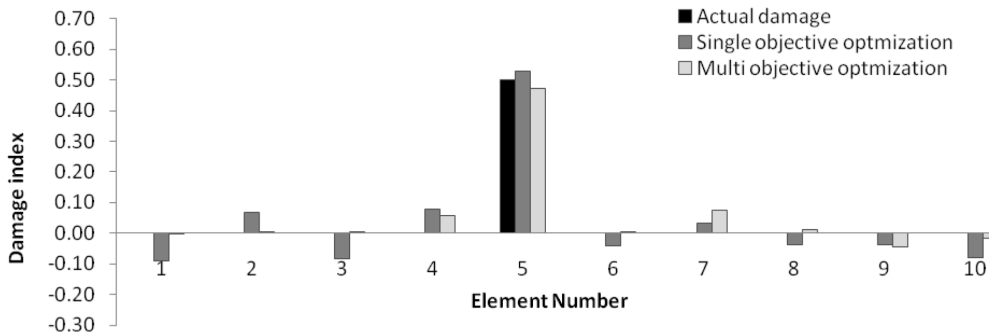
Following the conventional approach, an updated undamaged model was obtained using the objective function of Equation (4) and SOO. A total of 10 parameters were updated with 10 knowns, i.e. five frequencies and five MACs of the undamaged beam. After obtaining the updated undamaged model, the next step is to update the damaged model using the objective function of Equation (5). As the damage location was assumed to be unknown, all ten elements were updated in this case with 10 knowns in this case, i.e. five frequencies and five MACs of the damaged beam.

For MOO, a total of 20 parameters need to be concurrently updated, in which case 10 parameters belong to the undamaged beam and 10 parameters belongs to the damaged beam. Both Equation (4) and Equation (5) were concurrently used as two separate objective functions. The total number of knowns (frequencies and MACs) is 20 in this case and the number of unknowns (individual element MOIs) is 20. In each iteration, the updated FE model of the undamaged structure is further used for updating of the damaged structure. A typical Pareto front is shown in Figure 2 which considers both the objective functions equally important and identifies the non-dominated solutions. The front consists of those Pareto solutions for which there does not exist any solution which is better in both the objective functions simultaneously. Thus, the trade-off between both the objective functions can be explicitly decided by observing the Pareto front. In this research, the optimum solution is selected from the Pareto optimal set which minimizes the following expression:

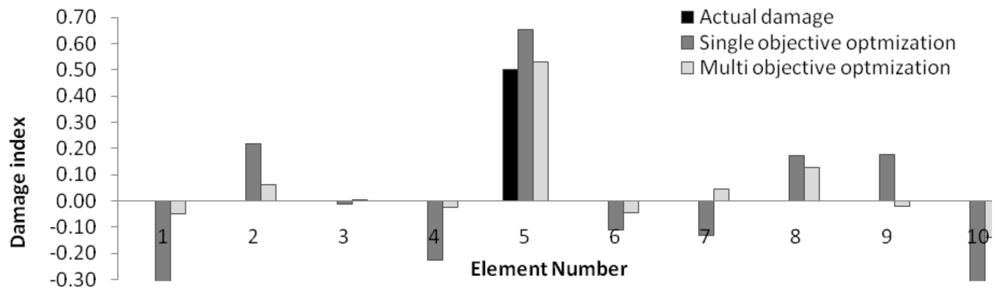
$$f = \sqrt{\Pi_{undam}^2 + \Pi_{dam}^2} \tag{6}$$

For assessment of damage severity, the updated undamaged model is compared with the updated damaged model for both SOO and MOO. Damage location and severity is estimated by subtracting the updated MOI ratios of the damaged model from that of the undamaged model. The actual damage index has been calculated as 0.5 for element No.5 in this study (as shown in Figure 1), which indicates a reduction of MOI of element No.5 from 0.9 to 0.4. The damage severity is estimated for all the elements and shown in Figure 3 along with actual damage for two noise levels considered, i.e. 5% and 20%.

From the results, it can be seen that damage estimation from MOO is more accurate than from SOO. For example, the damage index for element No. 5 (actual damage index = 0.5) is found to be 0.530 and 0.654 for the single objective case, whereas damage index for the same element is found to be 0.472 and 0.531 for the multi-objective case with 5% and 20% noise, respectively. For other elements, Figure 5 also indicates that MOI estimation was also markedly improved with the use of MOO. It can also be noticed that increase in the noise level also affects the damage severity estimates, i.e., with the increase in noise levels, the damage was also misestimated to a higher degree.



(a)



(b)

Figure 3. Damage severity estimation for element No. 5 : (a) 5% noise level, and (b) 20% noise level.

4 CONCLUSIONS

A method has been presented in this paper for damage estimation using modal data. Contemporary approaches to damage estimation compare model updating results for the undamaged and damaged structure acquired separately. The associated errors may propagate when the baseline undamaged model is subsequently used for damage estimation. A damage detection and estimation method which simultaneously updates the undamaged as well as the damaged structure model in a MOO process is presented herein. A numerically simulated simply supported beam has been used as an example problem and two noise levels of 5% and 20% have been added to the identified mode shapes to assess the performance of the proposed procedure. A better estimation of damage was obtained with this new technique as it effectively uses the experimental data of both the undamaged and damaged structure. It has been found that better results are obtained due to an increase in the available information for the undamaged and damaged states. Moreover this technique has proved to be less sensitive to experimental errors.

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