

# Long-Term Mode Shape Variations of Hagia Sophia with Environmental Factors

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**ABSTRACT:** Dynamic structural parameters depend on environmental factors such as temperature. Although cases documenting the effect of atmospheric conditions on natural frequency and damping are reported in the literature, studies on how mode shapes are changing over longer periods in the order of years are relatively rare. Mode shapes of historical buildings can be more complicated than of other structures. They are identified from vibration recordings obtained from structural monitoring systems. In this study, the mode shape variations of Hagia Sophia in Istanbul are examined using continuous recordings obtained between years 2013 and 2016. The modal frequencies of the structure are estimated by peak picking from Fourier amplitude spectra of 1-hour long windows. Mode shapes are calculated by Frequency Domain Decomposition (FDD) method. The variations in mode shapes are estimated for each window via the Modal Assurance Criterion (MAC) method, the Coordinate Modal Assurance Criterion (COMAC) and Enhanced Coordinate Modal Assurance Criterion (ECOMAC). Obtained MAC values for each window are combined to get the long term MAC variations and compared with those of environmental factors in search of any dependency between them. It was also possible to gain an understating of the range of variation of modal shapes over a time frame during which no known structural damage took place on the structure.

## 1 INTRODUCTION

Many researches have been done in the past to determine the dynamic behaviour of Hagia Sophia, which is included in the World Cultural Heritage List (Erdik et al. (1993), Durukal et al. (2003), Çaktı et al. (2016), Dar et al. (2018)). The long-term variation of modal frequencies and modal damping ratios of the structure were investigated by Çaktı et al. (2016) and Dar et al. (2018). In the present study, the change of mode shapes between 2013-2016 is investigated by different methods, which are the Modal Assurance Criterion (MAC), the Coordinate Modal Assurance Criterion (COMAC) and Enhanced Coordinate Modal Assurance Criterion (ECOMAC).

These methods can also be used for damage detection. However for its detection one needs to know the structural parameters that are related with damage, the limit values for different levels of damage and external factors affecting damage. Damage detection is not an easy task. For the prediction of structural damage level, the correlation of the past structural parameters with external factors such as temperature, humidity and so on, should be learned by artificial learning algorithms. In this way, the normal (undamaged) state of the structure under periodic external effects can be defined and delineated. In the case of mode shape, which is the parameter of interest in this paper, the natural tendencies associated with its variation needs to be understood. Unusual and beyond the limit changes in the mode shapes can be related to damage. The long-term



interaction of mode shapes of Hagia Sophia with atmospheric parameters is investigated herewith using the above-mentioned approaches.

The structural health monitoring system of Hagia Sophia consists of 9 3-way acceleration sensors and 4 tiltmeters. The locations of the acceleration sensors are shown in Figure 1. Tiltmeters are co-located with the acceleration sensors at the gallery level. Continuous acceleration data flowing in from the system corresponding to the time period between the years 2013 and 2016 were utilized in this study.

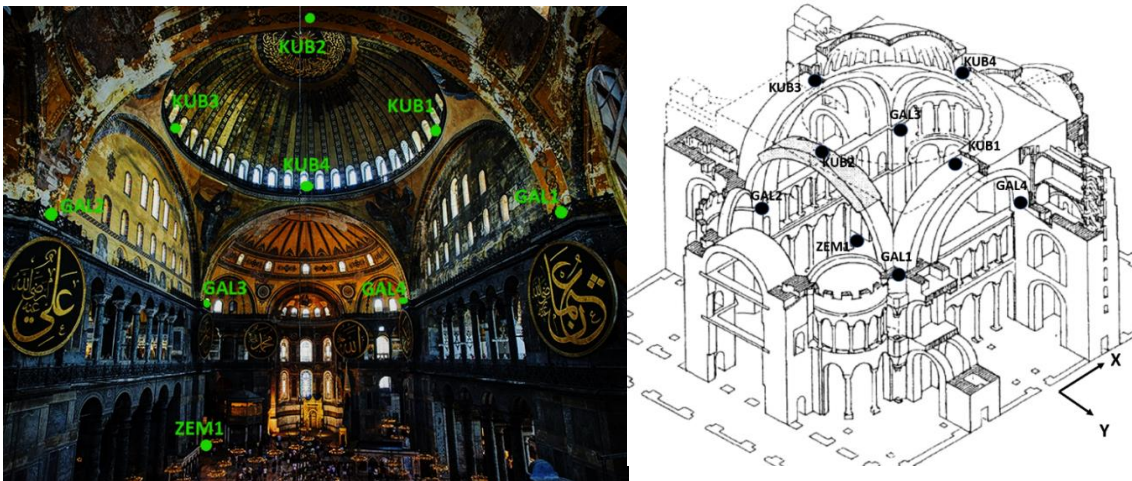


Figure 1. Locations of three-component accelerometric stations in Hagia Sophia. The isometric view is from Mainstone (2006).

## 2 METHODOLOGY

The methodology of this study can be grouped under two main headings: Big Data Analysis and Signal Processing. In the Big Data Analysis section, more than 500 Gb of raw data were classified, and the defective parts were identified and extracted. These records were then filtered and calibrated.

In the signal processing section, the mode shapes of the structure were obtained, and the time dependent changes were calculated. For this purpose, FDD, MAC, COMAC and ECOMAC methods are used. The whole process is shown in Figure 2.

In addition to occasional data absence in some channels of structural health monitoring systems, that might occur and need to be accounted for, there might be other issues such as time synchronization problems among stations, malfunctions in sensor or channels, and physical conditions of the stations. These problems need to be accounted for before further analyses, as they directly affect the results. For example, the sensor at station GAL4 has a time shift problem. When the system is reset, the time shift temporarily disappears, but then sets on. This situation poses a problem in the calculation of mode shapes. Another problem is that channel Y of station KUB3 was defective between January 2013 and November 2014. It was repaired later on. The records in this period were incorrect. Mode shapes were calculated by taking this situation into consideration. The final issue that had to be taken care of was related to instruments' positioning. The stations are positioned in the structures, such that individual channels' directions are aligned with principal structural axes. Between 2012 and 2014, sensor directions were off by 5 degrees in the Hagia Sophia system, which was corrected in December 2014. This situation had to be taken

care of during the signal processing as well. In the next section the details of the corrections and their effect on the results will be presented.

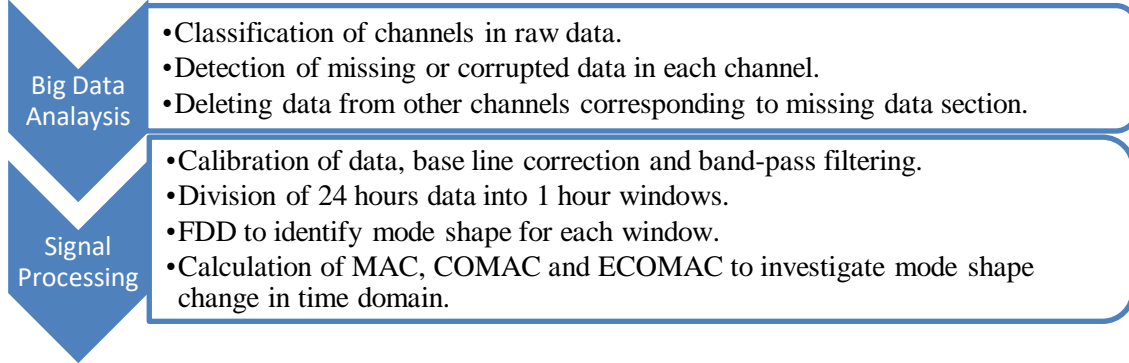


Figure 2. Flow chart of general methodology.

### 2.1 Frequency Domain Decomposition (FDD) Method

The FDD method is used for defining modes and determining the mode shapes. This method consists of two main stages. The first step is to obtain cross power spectral density functions (CPSD) and the second stage is the singular value decomposition. CPSD is the distribution of power per unit frequency and is defined as:

$$P_{xy}(\omega) = \sum_{m=-\infty}^{\infty} R_{xy}(m)e^{-j\omega m} \quad (1)$$

The cross-correlation sequence is defined as:

$$R_{xy}(m) = E\{X_{n+m}Y_n\} = E\{X_nY_{n-m}\} \quad (2)$$

where  $X_n$  and  $Y_n$  are jointly stationary random processes,  $-\infty < n < \infty$ ,  $-\infty < n-m < \infty$ , and  $E\{\}$  is the expected value operator. The second step is singular value decomposition of CPSD which can be defined as:

$$SVD(P_{xy})_{m \times m} = U_{m \times m} \Sigma_{m \times n} V_{n \times n} \quad (3)$$

$U_{m \times m}$  = Mode Shape Matrix

$\Sigma_{m \times n}$  = Mode Amplitudes

Mode Amplitudes are calculated for each modal frequency and maximum mode amplitudes corresponds to modal frequencies. Mode Shape matrix is calculated for each mode.

### 2.2 Mode Shape Correlation Methods

The Modal Assurance Criterion (MAC) is the most widely used criterion for vector correlation because of its simplicity. The MAC is the correlation coefficient of vector pairs in two vector sets  $\Phi_1$  and  $\Phi_2$  defined at the same system (Allemang (2002)). In this study,  $\Phi_1$  corresponds to measured mode shape at the first window in 1 January 2013 while  $\Phi_2$  corresponds to the following windows until 2017. The MAC is given by

$$MAC(j) = \frac{|\{\Phi_{j,t1}\}^T \{\Phi_{j,t2}\}|^2}{|\{\Phi_{j,t1}\}^T \{\Phi_{j,t1}\}| |\{\Phi_{j,t2}\}^T \{\Phi_{j,t2}\}|} \quad (4)$$

The Coordinate Modal Assurance Criterion (COMAC) has been developed to identify correlations of mode shapes that shows poor correlations in MAC (Lieven et al. (1988)). COMAC can be defined as:

$$COMAC(j) = \frac{\{\sum_1^N \{\phi_{j,t_1}\} \{\phi_{j,t_2}\}\}^2}{\{\sum_1^N \{\phi_{j,t_1}\}\}^2 \{\sum_1^N \{\phi_{j,t_2}\}\}^2} \quad (5)$$

The Enhanced Coordinate Modal Assurance Criterion (ECOMAC) is proposed to solve potential problems due to defective scaling, calibration or orientation of sensors (Hunt (1992)). ECOMAC can be computed as follows:

$$ECOMAC(j) = \frac{\{\sum_1^N \|\{\overline{\phi}_{j,t_1}\}\{\overline{\phi}_{j,t_2}\}\| \}^2}{2N} \quad (6)$$

$$\{\overline{\phi}_{j,t_1}\} = \{\phi_{j,t_1}\} / \|\{\phi_{j,t_1}\}\| \quad (7)$$

### 2.3 Time-Offset Correction Method

Station GAL4 has a time-shift problem as mentioned earlier. This time shift is not constant but displays a steady increase. Since the sensor has been reset many times over time, the time shift has continuously decreased and increased. The time shift at GAL4 with respect to other stations is shown in Figure 3, in which the accelerations at an arbitrary time are filtered around the first mode of the structure.

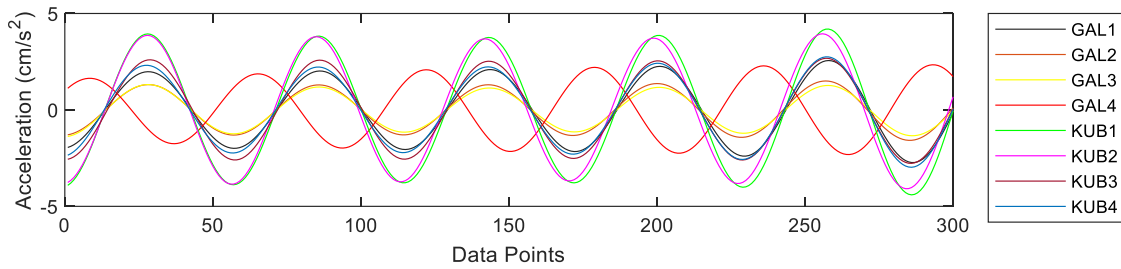


Figure 3. Time shift of station GAL4 with respect to other stations. Accelerations are filtered around the first mode of Hagia Sophia under free vibration.

Since the first mode of the structure is a harmonic movement, the time shift can be calculated using the cross-correlation method. The cross-correlation method can be explained as follows:

$$R_{xy} = \sum_{m=-\infty}^{\infty} \overline{f[m]g[m+n]} \quad (8)$$

Here “n” refers to the time delay. The cross-correlation function is calculated for different time delays in the negative and positive direction. The time delay closest to 1 indicates the highest correlation. In Figure 4, all stations except Gal 4 reach maximum correlation at n = 0. However, GAL4 station reaches maximum correlation at n = 250. This shows that the GAL4 station is lagging behind the other stations for 250 data points.

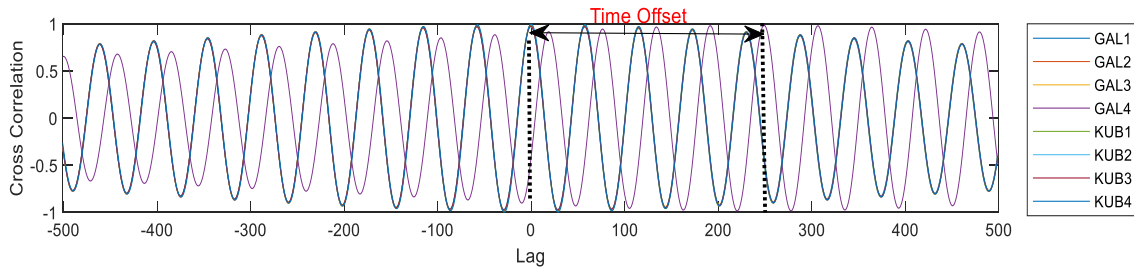


Figure 4. Cross-Correlation Function of Stations.

In the analysis carried out in this study, it was observed that time shift calculations could not be reliable in the windows below 90 000 data points. Therefore, 1-hour windows (360 000 data points) are preferred in this study. When cross-correlations are calculated for each window, the



time-dependent change of time-offset can be calculated. Figure 5 shows the change of time shifts at station GAL4 in 2015. As seen, the time offset increases linearly and becomes zero when the system is reset.

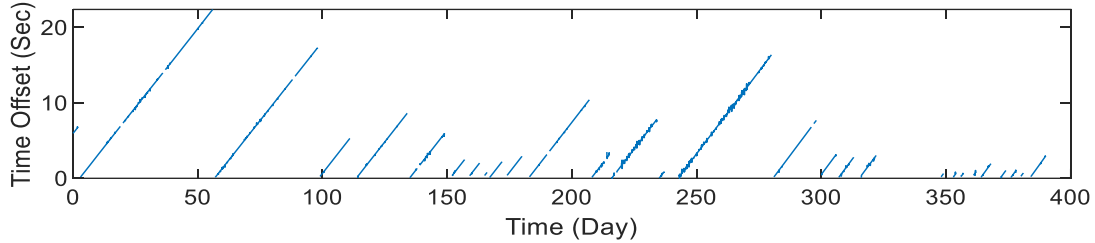


Figure 5. Time-Offset variation of GAL4 station in 2015.

### 3 RESULTS

Correlation of long-term mode shape patterns was calculated for 7 stations in the first stage by excluding GAL4. For each window, using the MAC, COMAC, and ECOMAC methods, the time-dependent changes of the first (X direction) and second (Y direction) mode shapes between years 2013-2016 were calculated. For all mode shape correlation calculations, the first 1-hour window on 1 January 2013 was taken as a reference and the correlation of other windows with the first one was calculated. The first and second mode shape changes calculated using three different methods are shown in Figure 6 and Figure 7, respectively.

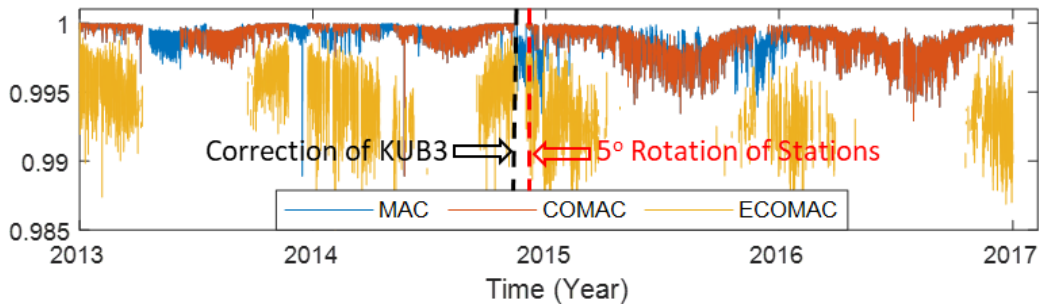


Figure 6. Variation of the first mode shape between 2013-2016.

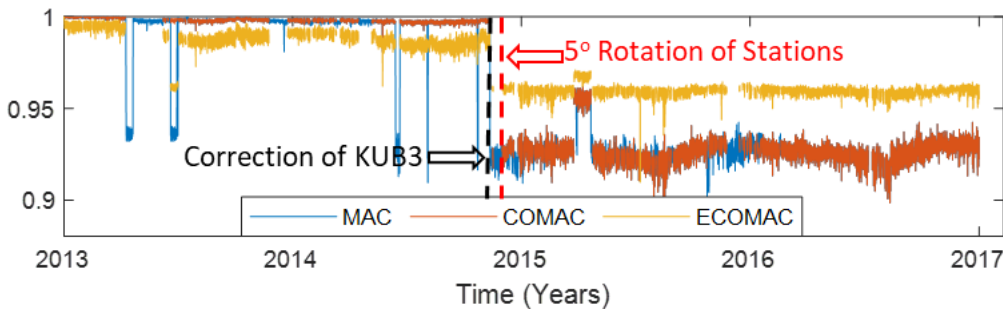


Figure 7. Variation of the second mode shape 2013-2016.

There are two important reasons that may explain the results of the analysis displayed in the figures. The first reason is the replacement of KUB3 on 15 November 2014, which had a malfunctioning channel (Y), with a new one. The second is that the all stations were rotated by 5 degrees to align them with structural axes on 10 December 2014. The effect of this change is not seen in the first mode in the X direction. However, it is clearly seen in the second mode shape in the Y direction. The change is about 8% in MAC and COMAC, and %4 in ECOMAC.

Rotation of the sensors by 5 degrees has less than 0.5% effect in both modes. This is because all sensors are rotated by the same amount. For this reason, although the displacement of the measured structure has changed, the mode shapes have not changed significantly since the displacements are normalized in MAC, COMAC and ECOMAC methods.

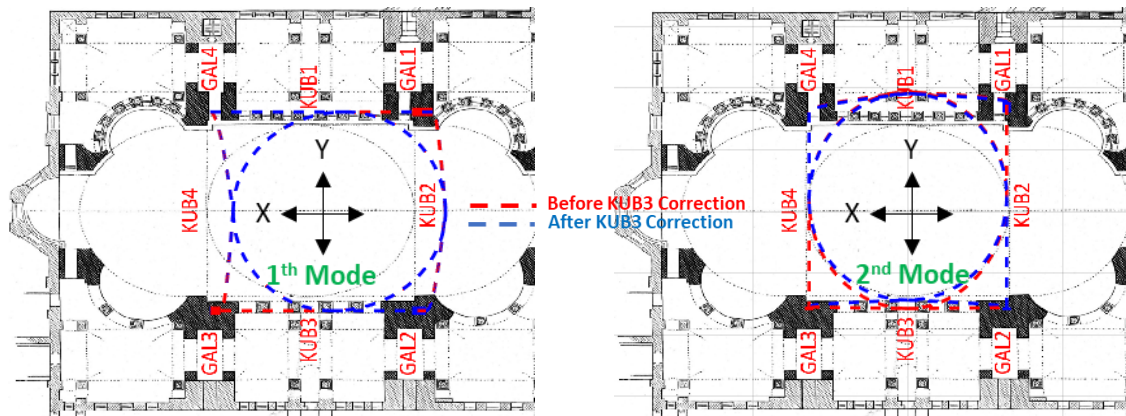


Figure 8. The first (right) and second (left) mode shapes before and after KUB3 correction.

The effect of changes in sensors on the analysis results is shown by calculating the three-dimensional mode shapes of the structure. Figure 8 shows the effect of the change in the KUB3 station for the first and second modes, respectively. The blue line shows the mode shape before the replacement and the red line shows the mode shape after the replacement. As can be seen, there was no noticeable change in the first mode. However, in the second mode, KUB3 station was observed to displace more after the replacement. This significantly affects the MAC, COMAC and ECOMAC results calculated for the second mode. In both modes, the effect of angular rotation on the mode shape is not visible. This shows that the small deviations in the direction of the sensors will have limited effect on the mode shape calculations.

### 3.1 Atmospheric Effects on Long-Term MAC

Past studies have shown that changes in atmospheric conditions change the modal frequency and modal damping ratio of structures. In this study, it was observed that the mode shapes were temporarily affected by changes in the atmosphere conditions (Figure 9-11). The increase in air temperature increases the modal frequency, decreases the modal damping ratio and accordingly changes the mode shape. The instantaneous changes in the wind speed may also affect the calculated mode shape momentarily, but this effect cannot be seen in the present analysis, because the mode shape is calculated for one-hour windows. A correlation has been observed between the relative humidity and the change in the mode shape, but it is difficult to determine how much it affects the mode shape because relative humidity is a highly dependent parameter to the air temperature.

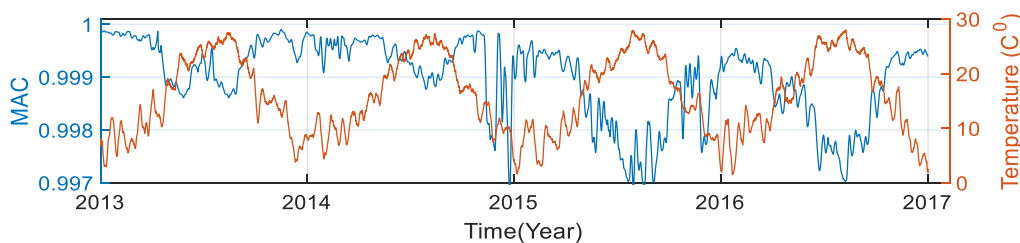


Figure 9. Effect of air temperature on mode shapes variations in the first mode.

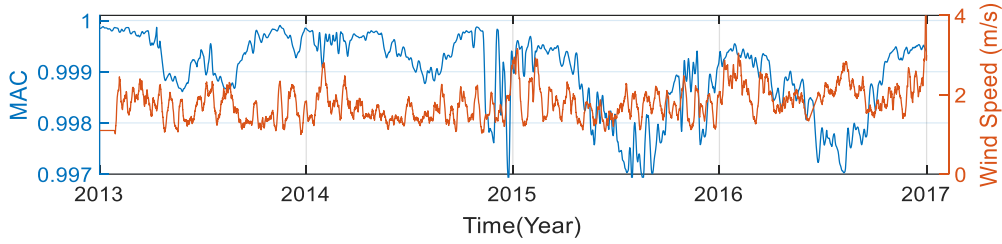


Figure 10. Effect of wind speed on mode shapes variations in the first mode.

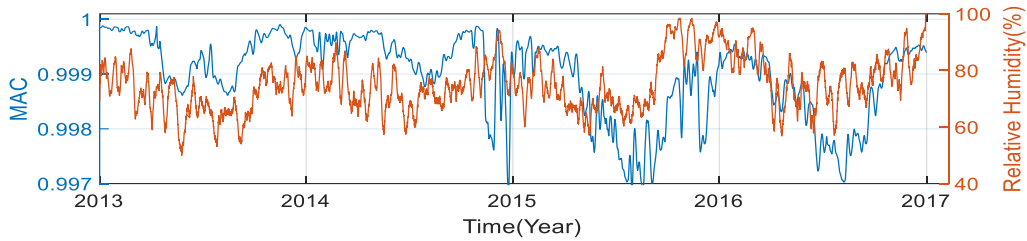


Figure 11. Effect of relative humidity on mode shapes variations in the first mode.

### 3.2 Influence of Time-Shift on MAC

The effect of the time shifts at GAL4 on mode shapes in long-term is estimated for the first mode and shown in Figure 12. As the time shift is constantly increasing, it is constantly moving away from the mode of the structure and then moving closer again. As a result, the MAC value ranges from 0.92 to 1. The time offset was eliminated with the help of the cross-correlation method and the results were compared with the MAC calculated with and without GAL4. As can be seen in Figure 13, there is less than 0.2% difference between the MAC values obtained from the 7-station mode shape and the 1-corrected/ total 8-station mode shape.

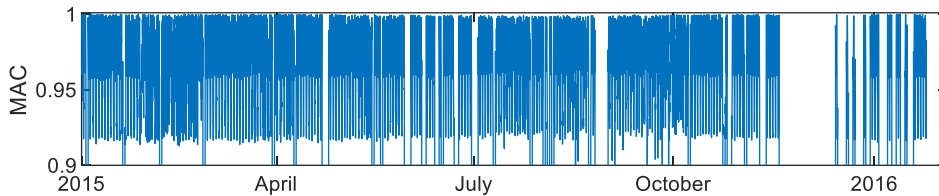


Figure 12. The effect of the time shifts at GAL4 on the long-term mode shape for the first mode.

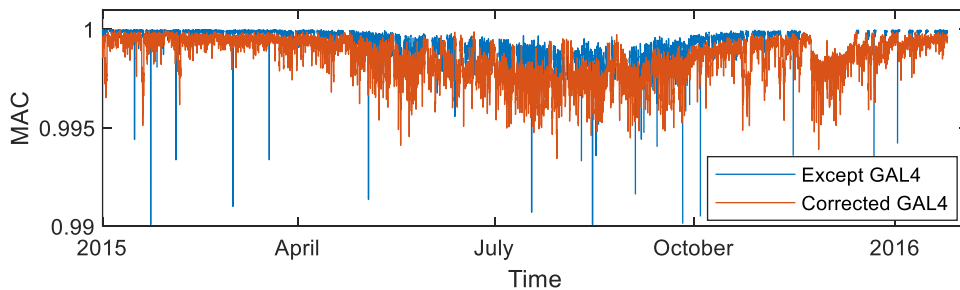


Figure 13. The MAC values obtained from the 7-station mode shape and the 1-corrected total 8-station mode shape on mode shape 1.

#### 4 CONCLUSIONS

It was found that in the four-years of observation between 2013 and 2016, the MAC values of Hagia Sophia have displayed a periodic variation. The range of variation was 99.6% - 100%. The length of one cycle was about one year. No permanent shift in the MAC values, that could be correlated with some structural defect was observed. The range of variation is very narrow, but its dependence on atmospheric conditions, particularly on temperature is clear. The technical problem in channel Y of station KUB3 affected the mode shape correlations in the second mode of the structure by 8%, but the mode shape was consistent in itself after this issue was resolved. The effect of the 5° rotation in the sensor direction on the mode shapes was very limited. The time offset problem at GAL4 station was solved by using the cross-correlation method and the results were consistent with the calculated MAC values without GAL4.

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