

A comparison of greedy and global searches for measurement-system design in bridge load testing

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ABSTRACT: Due to conservative design models and safe construction practices, infrastructure often has significant yet-unknown reserve capacity that greatly exceeds code requirements. Reserve-capacity assessments lead to better asset-management decisions through either avoiding unnecessary replacement or lowering maintenance expenses. Field measurements have the potential to improve the accuracy of model predictions. To fulfil this potential, measurements, must be associated with an adequate structural-identification methodology. Error-domain model falsification is an intuitive model-based methodology that explicitly represents systematic uncertainties that are typically associated with structural models. Additionally, model-updating outcomes depend on the design of the measurement system. Engineers usually select sensor types and place sensors based on experience and signal-to-noise estimations. The development of more rational strategies for measurement-system design has recently received research attention. Quantitative sensor-placement strategies differ either in the objective function for sensor placement or in the optimization algorithm used. This study presents a comparison of greedy-search (hierarchical) and global-search (such as genetic algorithm or Probabilistic global-search Lausanne) methodologies in terms of joint-entropy evaluations, recommended sensor configurations and qualitative characteristics using a full-scale test study, the Rockingham Bridge (Australia). Results show, for low number of sensors, that global-search algorithms only slightly over-perform the greedy-search algorithm in terms of information gain. However, this is at the expense of a longer computational time compared with greedy search. Nevertheless, global-search strategies provide other advantages such as finding multiple near-optimal sensor configurations. These advantages are illustrated using the full-scale bridge case.

1 INTRODUCTION

Countries are experiencing infrastructure-capacity challenges due to population growth, increasing urbanization and ageing existing civil assets. Due to conservative design models, infrastructure often has unknown amounts of reserve capacity beyond code requirements. The reserve-capacity assessment of existing infrastructure has the potential to improve decision-making on asset management. However, this task is challenging due to the difficulties in modelling infrastructure behavior (Catbas et al. 2013).

The aim of model-based structural identification is to use in-situ measurements to improve behavior model predictions. (Smith 2016) has shown that usual model calibration methodologies, such as residual minimization or traditional Bayesian model updating, cannot be justified for large-scale structures because they often result in biased identification. To account for systematic uncertainties, (Goulet and Smith 2013) presented a new model-based methodology, called error-domain model falsification (EDMF). In this approach, a population of model instances is

generated with unique sets of model-parameter values. Field measurements are used only to falsify wrong instances, when model-instance predictions significantly differ from measurements. EDMF was shown to be more robust than traditional methods when wrong initial model-class assumptions are made (Pasquier and Smith 2016).

Regardless of the approach used for model updating, a rational sensor-placement methodology has the potential to enhance structural identification. (Papadimitriou 2004) showed that the information entropy could be used as a criterion to discriminate possible sensor locations. A location with a high entropy value in model predictions (Robert-Nicoud et al. 2005a) or offering high entropy reduction of parameter values (Argyris et al. 2017) is considered as a good location. To account for mutual information between sensors (Bertola et al. 2017a; Papadopoulou et al. 2014) used the joint-entropy objective function for sensor placement.

Due to the exponential number of sensor configurations with respect to the number of sensor locations, all combinations cannot be evaluated in a reasonable computational time. An optimization strategy is thus required. Greedy optimization algorithms, such as sequential algorithms with a forward or backward strategy, were used to reduce computational time (Kammer, 1991; Papadimitriou, 2004). As greedy search may lead to a suboptimal sensor configuration, other studies selected a global-search strategy such as genetic algorithm (Chow et al. 2011), particle swarm optimization (Kukunuru et al. 2010) or Probabilistic Global-Search Lausanne (Kripakaran and Smith 2009) among many others.

Despite the large variety of optimization algorithms used in sensor-placement studies, only a few performance comparisons of sensor configurations were performed see (P. Kripakaran et al. 2007). Additionally, the joint-entropy objective function, enhancing the sequential sensor placement using the hierarchical algorithm (Bertola et al. 2017b), has not been compared with global search strategies.

This study compares the performance of greedy- and global-search optimization algorithms for sensor placement, in terms of qualitative characteristics, quantitative information-gain metric and recommended sensor configurations. The study is organized as follows. Section 2 presents the error-domain model falsification framework for model updating. Sections 3 describes sensor-placement strategies used in the present study. Afterword, Section 4 presents the full-scale bridge case study and results are discussed.

2 SYSTEM IDENTIFICATION THROUGH MODEL FALSIFICATION

Error-domain model falsification (EDMF) is an intuitive model-based structural-identification methodology (Goulet and Smith 2013). The method compares field measurements with predictions of an initial set of possible model instances to identify plausible models.

First, the initial population of model instances θ_k is generated by sampling unknown parameter values of behavior models $\theta_k = [\theta_1, \theta_2, \dots, \theta_n]^T$. Initial parameter ranges are defined using prior knowledge on the structure. Then, at each measurement location $i \in \{1, \dots, n_y\}$, model-instance predictions are compared with field measurements. At a sensor location i , the model prediction $g_k(i, \theta_k)$ and the field measurement \hat{y}_i differ since model-prediction uncertainties $U_{i,g}$ and measurement uncertainties $U_{i,\hat{y}}$ are unavoidable. Nevertheless, they are linked to the true behavior R_i using the following equation:

$$g_k(i, \theta_k) + U_{i,g_k} = R_i = \hat{y}_i + U_{i,\hat{y}} \quad \forall i \in \{1, \dots, n_y\} \quad (1)$$

Eq. (1) is rearranged Eq. (2), where modeling and measurement uncertainties ($U_{i,g}$ and $U_{i,\hat{y}}$) are combined in a unique source $U_{i,c}$. The difference between the model prediction and the field measurement at a sensor location i is called the residual r_i .

$$g_k(i, \theta_k) - \hat{y}_i = U_{i,c} = r_i \quad (2)$$

Model instances for which residuals exceed threshold bounds, according to a confidence level fixed at 95% of the combined uncertainty, are falsified. Model instances for which residuals do not exceed threshold bounds at each sensor location are considered as plausible and are included in the candidate model set (CMS). Model instances belonging to the CMS are equivalently likely (Robert-Nicoud et al. 2005b). If all initial model instances are falsified, this often means that the selected model class is incorrect. Complete-falsification cases help avoid wrong parameter-value identifications and help detect wrong initial assumptions, showing an advantage of EDMF compared with other structural-identification methodologies (Pasquier and Smith 2016).

3 SENSOR-PLACEMENT FRAMEWORK

The aim of a sensor-placement strategy is to identify optimal sensor locations when a limited knowledge of model-parameter values is available. Figure 1 presents the framework of a sensor-placement strategy. Once the numerical model is built and the model class is selected, measurement predictions from a population of model instances, generated using a sampling strategy, are initial inputs of model-based sensor-placement methodologies. Then, the objective function and the optimization algorithm must be selected to identify the optimal sensor configuration with respect to the number of sensors.

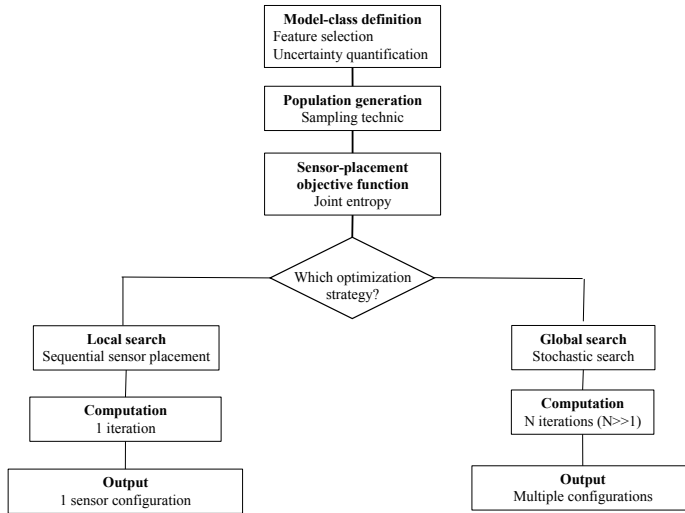


Figure 1 Sensor-placement framework.

3.1 Sensor-placement objective function – joint entropy

In this section, the objective function used to define the optimal sensor configuration is presented. The information entropy, or Shannon entropy, was introduced as a sensor-placement objective function for system identification by (Papadimitriou 2004). In EDMF framework, range of prediction at each sensor location i is divided in intervals based on the combined uncertainty $U_{i,c}$ (Eq. (2)). The probability that model instance output $y_{i,j}$ fall inside the j^{th} interval among $N_{I,i}$ intervals is equal to $P(y_{i,j}) = m_{i,j} / \sum m_{i,j}$, where $m_{i,j}$ is the count of model instances in the j^{th} interval. The information entropy $H(y_i)$ is evaluated for location i as:

$$H(y_i) = - \sum_{j=1}^{N_{I,i}} P(y_{i,j}) \log_2 P(y_{i,j}) \quad (3)$$

Papadopoulou et al. (2014) proposed joint entropy as a new sensor-placement objective function, considering the redundancy of information gain between sensor locations. The joint entropy evaluates the information entropy between sets of predictions while taking into account the mutual information between them. For a set of two sensors, it is defined as:

$$H(y_{i,i+1}) = - \sum_{k=1}^{N_{i,i+1}} \sum_{j=1}^{N_{i,i}} P(y_{i,j}, y_{i+1,k}) \log_2 P(y_{i,j}, y_{i+1,k}) \quad (4)$$

where $k \in \{1, \dots, N_{i,i+1}\}$ and $N_{i,i+1}$ is the maximum number of prediction intervals at the $i+1$ location and $i + 1 \in \{1, \dots, n_s\}$ with the number of potential sensor locations n_s . The joint entropy is less than or equal to the sum of the individual information entropies of sets of predictions. Eq. (5) presents the joint entropy of two sensors, where I is the mutual information between sensor i and $i+1$.

$$H(y_{i,i+1}) = H(y_i) + H(y_{i+1}) - I(y_{i,i+1}) \quad (5)$$

3.2 Optimization algorithms

In this section, the optimization strategies compared in this study are presented. First, the greedy-search methodology is shown in Section 3.2.1. Then, two global-search methodologies are presented in Sections 3.2.2 and 3.2.3

3.2.1 Greedy search

The hierarchical algorithm (Papadopoulou et al. 2014) is a sequential algorithm with a forward strategy. In a sequential search, once a sensor location is selected, this choice is not reevaluated in subsequent sensor placements. The hierarchical algorithm organizes model instances in a tree structure. The initial model set is the root and branches represent possible subdivision of the model set using measurement at a specific location. For each branch, the joint entropy is evaluated and the location with the largest joint-entropy value is selected. This value represents the sensor location that has the highest potential in dividing the existing subsets of model instances into smaller subsets. The process is repeated by adding subsequent sensors to the sensor configuration until all possible sensor locations are included in the sensor configuration.

3.2.2 Global search – Genetic algorithm

The genetic algorithm (GA) is a well-known global-search optimization introduced by (Holland 1992). This evolutionary algorithm mimics biological processes of reproduction and selection. An initial population is generated randomly and each candidate solution, called chromosomes, is evaluated through an objective function (fitness value). Chromosomes with largest fitness-function values are selected similarly as natural selection and evolved through “mutations” to maximize locally the fitness function. In order to find the global optimum of the objective function, “crossovers” on the chromosome population occur randomly to generate a new population. Objective-function evaluations of the new chromosomes are then evaluated and process of natural selections are repeated. The optimization process stops when the maximum number of new-population generation or maximum objective-function value is reached

3.2.3 Global search – Probabilistic global-search Lausanne

Probabilistic Global-Search Lausanne (PGSL) is a direct search algorithm that uses global sampling to find the optimum of an objective function (Raphael and Smith 2003). PGSL has been successfully applied to optimization cases involving high non-linear objective functions with multiple local optima, such as measurement-system design (Kripakaran and Smith 2009). PGSL focuses search of optimal solutions around sets of good solutions. The search space is sampled by means of probability density functions (PDFs) defined over the entire space. As search progresses, PDFs are updated in order to generate new solutions in regions containing good solutions with higher probability. The search space is gradually narrowed down until convergence is achieved. The optimization process stops when the maximum number of iterations or maximum objective-function value is reached.

4 FULL-SCALE CASE STUDY

4.1 Bridge presentation

The full-scale case study is the Rockingham bridge, located near Perth (Australia). This pre-stressed concrete bridge is composed of 8 beams over three spans of approximately 14, 24 and 13 meters. A static load test was performed involving 2 trucks of 80 tons placed by engineers to maximize the deflection of the second span. Figure 2 presents a photograph of the bridge and the ANSYS-WORKBENCH numerical model. The sensor configuration consisting of 6 strain gauges (Figure 2B, squares) was installed based on engineering judgement. This configuration is discussed below. Possible sensor locations every two meters were taken into account (Figure 2B, triangles) for the sensor-placement optimization (Section 4.2).

The following model parameters were identified as having the most influence on predictions: The Young moduli of site-cast concrete of the deck E_{con} [GPa] $\theta_1 \in [25,45]$ and of the precast concrete of the beams E_{pre} [GPa] $\theta_2 \in [30,50]$. Model-parameter intervals were chosen based on engineering judgement. The initial model set consists of 300 instances, generated using the Latin Hypercube Sampling. Model-class and measurement uncertainties are presented in Table 1 and uniform distributions between bounds are assumed. They are estimated based on engineering judgment and sensor-supplier information. Since solid elements are used to build the FE model, the model is expected to have a stiffer behavior compared to the bridge up to 15 %. However, due to model simplifications such as transversal beams or absence in the model of non-structural elements, the model behavior may present a softer behavior up to 5 %.

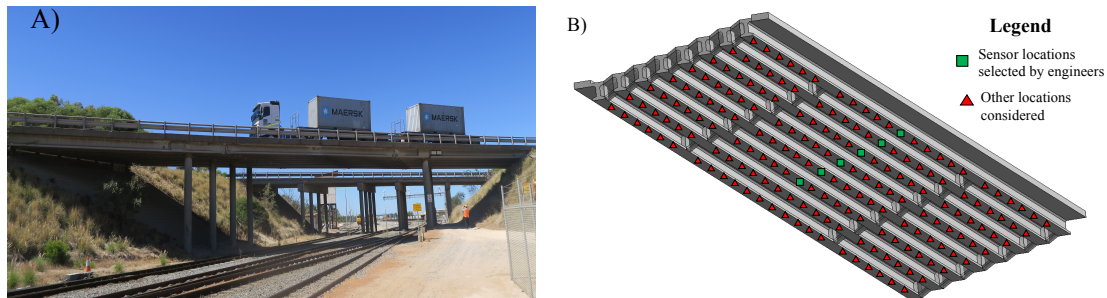


Figure 2 A) Bridge photograph; B) numerical model with possible strain locations.

Table 1 Modelling and measurement uncertainties.

Uncertainty source	Uncertainty bounds	
	Min	Max
<i>Model uncertainties</i>		
Model simplifications (%)	-5	15
Mesh refinement (%)	-1	1
Additional uncertainty (%)	-2	2
<i>Measurement uncertainties</i>		
Sensor accuracy (%)	-1	1
Repeatability (%)	-2	2
Sensor resolution ($\mu\epsilon$)	-3	3

4.2 Comparison of optimization strategies for sensor placement

In this section, results from greedy and global strategies for sensor-placement are compared. Table 2 presents qualitative characteristics of both strategies. While the greedy search in a single

iteration give a sensor configuration as function of number of sensors, the global search strategy must be performed independently for any number of sensors. Additionally, global search requires a much larger computational time than greedy search for a single iteration. Greedy search may often lead to suboptimal solution, especially for a low number of sensors. Nevertheless, to guarantee to find a “good” solution (defined as at least equally performing as configurations proposed by greedy algorithm in terms of information gain), the global-search strategy must run a sufficient number of iterations. This minimum number of iterations is function of the number of variables (i.e. sensor to place) but also to the ratio between informative and non-informative locations (Goulet 2012). If a sufficient number of iterations is performed, the global-search strategy usually recommends several equally-performing alternatives, providing choices to asset-managers in terms of sensor configurations. Conversely, the greedy search suggests only a single sensor configuration but provides a ranking of sensor locations that may be useful to prioritize the sensor installation.

Table 2 Qualitative characteristics of global-search and greedy-search strategies for measurement-system design.

Characteristic	Global search	Greedy search
Low computational time	X	✓
Near-optimal solution	✓	?
Multiple alternatives	✓	X
Ranking of sensor locations	X	✓

Figure 3 presents the comparison of optimization algorithms in terms of expected information gain (joint entropy) as function of number of sensors. Expected information gain of the sensor configuration installed by engineers is also presented. Global search strategies as PGSL and GA were performed for 2, 4, 6, 10 and 16 sensors, using 3200 iterations, which corresponds to 20 times the largest number of variables. First, expected information gains of all optimization strategies are larger than the expected information gain from sensor configuration selected by engineers, showing that a rational sensor-placement strategy beyond engineering judgement can enhance structural identification. For sensor configurations involving 2 to 4 sensors, global-search strategies slightly outperform the greedy-search strategy, while for 6 to 16 sensors they underperform the greedy-search strategy in terms of expected information gain. Therefore, global-search strategy does not always provide good sensor-configuration recommendations. A greedy search is thus recommended as optimization algorithm, except if several alternatives of sensor configurations are required. Comparing the global search strategies, PGSL outperforms GA for all number of sensors in the sensor configuration.

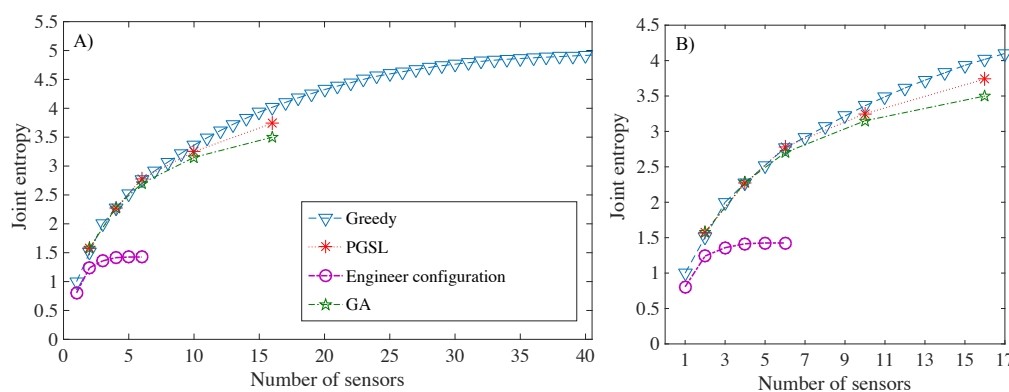


Figure 3 A) Joint entropy as function of number of sensors for several optimization strategies and sensor configurations installed by engineers. B) Zoom-in of the figure on the left.

Figure 4 presents sensor configurations recommended by greedy- and global-search algorithms. Results are compared for 2 (Figure 4A) and 4 (Figure 4B) sensors in the sensor configuration. Only sensor configurations that outperform the greedy search in terms of information gain are displayed. Globally, several sensor configurations provide similar information gain, showing that the objective function of joint entropy seems to have several equivalent local optima. Concerning 2-sensor configurations, solutions involve a sensor close to the support and another near midspan, showing that locations with large signal-to-noise ratio are preferred. To avoid redundancy in information gain, sensors are proposed on several beams. However, almost all beams are selected in at least one sensor configuration, showing no particular preference on beam selections. Concerning 4-sensor configurations, only three configurations of global-search strategies slightly outperform the configuration proposed by the greedy algorithm. For all strategies, sensor configurations consist of 3 sensors next to the supports and 1 sensor close to midspan and sensors are placed at least on two beams.

A near-optimal sensor configuration involves selecting locations with large signal-to-noise ratio but also placing sensors transversally on the bridge. Engineers selected sensor locations only at mid-span (Figure 2B). Although sensor locations have large signal-to-noise, this choice is suboptimal when compared to sensor configurations proposed by sensor-placement algorithms.

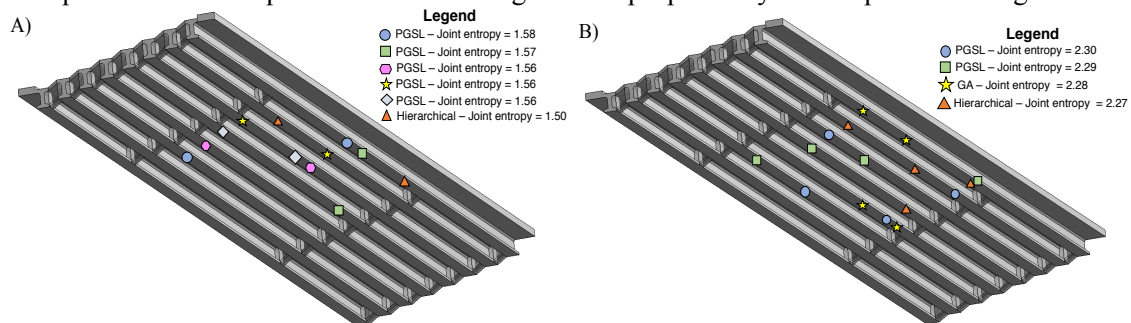


Figure 4 Sensor configurations proposed by greedy- and global-search algorithms. A) Configuration of 2 sensors; B) Configuration of 4 sensors.

5 CONCLUSIONS

A static load test is studied for sensor placement on a full-scale bridge in Rockingham. This study compared greedy- and global-search algorithms for sensor placement. Both types of optimization algorithms provide measurement systems that can enhance structural identification, when compared with the measurement system selected by engineers.

The greedy-search algorithm best supports engineers for measurement-system design when information-gain metric and computational time are taken into account. Nevertheless, global-search algorithms provide several good configurations and thus they provide engineers with a flexible choice in terms of sensor configuration.

A more complete comparison should include performance criteria such as monitoring costs, sensor installation and robustness of information-gain to sensor failure. This has been proposed in related work, see (Bertola et al. 2019).

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