Possibilities to Enhance Tomography Imaging of Concrete Structures by the Full Waveform Inversion

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ABSTRACT: It is a state of the art to test concrete specimen with ultrasonic-based methods. However, due to the density and the required high resolution, the testing is done with high-frequency waves which cannot penetrate the structure in depth. Adopting techniques used, e.g., in geotechnical applications, a model-based inversion using higher wavelengths is proposed, which is the full waveform inversion. This approach is however non-linear, time-consuming and highly ill-posed. Thus, further developments are necessary to make the technique applicable to structures like dams, dikes or similar. Among these developments are further accelerations of the so-called forward codes by a more efficient implementation of the wave equation. To handle with the ill-posedness, regularization techniques need to be applied and tested in order to reduce the risk of incorporating too many and misleading artifacts in the resulting images. This, e.g., can be done by the consideration of a priori knowledge of the material properties inside a structure which could be taken from already well-established inspection methods. Some first results, mainly based on synthetic data, will be presented to show the capabilities of the model-based approach.

1 INTRODUCTION

The importance of life-cycle management is increasing steadily over the last years. In the case of engineering structures it becomes inevitable and since the majority of these structures is still in use, the damage detection should be done non-invasively. Hence, the full waveform inversion is an useful tool to identify material parameters of soil and structures probing them with seismic waves. Nevertheless, the investigation of huge structures is complex, time-consuming and also costly. Therefore it is recommended to optimally design the experiments, e. g. for damage detection, to increase the quality of the results and also to save time and costs. The content of this paper is to investigate the influence of the (optimal) sensor positions on the quality of the results of the full waveform inversion.

Firstly, the principles of full waveform inversion (FWI) and optimal design of experiments (OED) are presented. Their use in the case of a two-dimensional time-domain finite-difference scheme for FWI is explained and applied on a simplified dam structure using a free software for FWI. A genetic algorithm is used in order to find the optimal experimental design by searching for the minimal difference of the model parameters between the original/true model and the models gained by the FWI. The numerical results for a non-optimal design, the initial guess and the optimal design are shown and compared. Further on, the results are discussed and suggestions for further investigations are made.

2 THEORY AND METHODS

2.1 Full Waveform Inversion (FWI)

It is possible to gain information about material parameters from probing it with seismic waves. Here, the transmission, reflection, refraction and diffraction data is used in order to identify fault areas in the structure and/or soil.

Generally, seismic waves are divided into surface and body waves, where only the latter are considered in this approach. Again, there are two parts describing the body waves. The primary wave (P-wave) is a compression wave, which travels through all media with a change in the velocity for different materials. The other part is described by the secondary wave (S-wave) and is a shear wave, which only exists in solids.

The FWI is solving an inverse problem where the residual energy between the experimental data and the numerical data of the underlying model is minimized as stated in Tarantola (1984). The numerical model in two dimensions is formed by using the elastic wave equation formulated in 2D (x-z-plane) as in Köhn (2011)

$$\begin{aligned} &\rho(x,z)\frac{\partial v_{\boldsymbol{i}}}{\partial t} = \frac{\partial \sigma_{ij}}{\partial x_j} + f_i\,, \\ &\frac{\partial \sigma_{ij}}{\partial t} = \lambda(x,z)\frac{\partial \theta}{\partial t}\delta_{ij} + \mu(x,z)\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \\ &i = x,z; \quad j = x,z; \quad + \text{boundary conditions and initial conditions}\,, \end{aligned} \tag{1}$$

where $\rho(x,z)$ is the density, v_i are the particle velocities, t stands for time, σ_{ij} denotes the stress tensor components, f_i are the directed body forces, $\lambda(x,z)$ and $\mu(x,z)$ are the Lamé parameters and δ_{ij} is known as Kronecker's delta. In this paper finite differences are used to solve the elastic wave equation on a discretized domain, where perfectly matched layers as in Berenger (1994) are used to prevent the wave reflection on the boundaries of the domain.

For the utilization of the FWI it is necessary to gain experimental data $\boldsymbol{u}^{\text{exp}}$ and to develop an appropriate initial numerical model containing numerical data $\boldsymbol{u}^{\text{num}}$. The inversion procedure is then controlled by minimizing the cost function

$$C_f(\boldsymbol{m}) = \|\boldsymbol{u}^{\text{num}}(\boldsymbol{m}) - \boldsymbol{u}^{\text{exp}}\| \tag{2}$$

in the sense of least squares leading to a comprehensive model of the structure of interest as in Alalade et al. (2018). In each iteration step the wave equation is solved for each excitation source and frequency to obtain $\boldsymbol{u}^{\text{num}}$. Afterwards, the residuals

$$\delta\boldsymbol{u}(\boldsymbol{m}) = \boldsymbol{u}^{\text{num}}(\boldsymbol{m}) - \boldsymbol{u}^{\text{exp}} \tag{3}$$

are calculated and they are backpropagated from the receiver positions to generate the adjoint wavefield. The model parameters are updated simultaneously by using a Quasi-Newton method under consideration of the acquired $\delta\boldsymbol{m}$ while a regularization is introduced by an early stopping criterion.
Here, the model parameters are formed by the density ρ, the P-wave velocity

$$v_{\text{p}} = \sqrt{\frac{\lambda+2\mu}{\rho}} \tag{4}$$

and the S-wave velocity

$$v_s = \sqrt{\frac{\mu}{\rho}}\,, \tag{5}$$

where the latter two are calculated by the density and the Lamé parameters.

2.2 Optimal Design of Experiments (OED)

The aim of optimally designing experiments is to increase the quality of the results and also to improve the damage detection. By reducing the number of sensor positions to an optimum it is possible to save time for installation and data analysis, but also to diminish the costs.

As for the state of the art it is discriminated between well-posed and ill-posed problems. The former are usually solved by applying the Fisher information matrix (FIM, cf. Uciński (1999)), which means either maximizing the parameter sensitivities or minimizing the parameter uncertainties. These kind of problems lead to unbiased parameter estimates, which implies that only random measurement errors can be considered, but not systematic ones. Often strategies for well-posed problems are applied on ill-posed problems as well. Nevertheless, novel approaches are often using the mean-squared error (MSE) or similar measures for finding optimal experimental designs (cf. Bardow (2008), Haber et al. (2008), Haber et al. (2010), Lahmer (2011), Schenkendorf et al. (2009)).

Since the model parameters identified by the FWI depend on the positions of the sources and receivers, which were used during the FWI process, the optimal experimental design can be calculated. Therefore, the root-mean-squared error (RMSE)

$$\mathrm{RMSE}(\boldsymbol{x}) = \|\boldsymbol{m}(\boldsymbol{x}) - \boldsymbol{m}^*\|_2 \tag{6}$$

between the reconstructed parameters $\boldsymbol{m}(x)$ depending on the sensor positions $\boldsymbol{x}$ and the reference parameter values $\boldsymbol{m}^*$ is used as the objective function. When the RMSE is minimal the optimal sensor positions for the sources and receivers are found.

3 APPLICATION

A simplified model of a dam structure as shown in Figure 1 serves as the application example. The FWI problem is solved in a two-dimensional time domain finite-differences scheme. The size of the domain is 210 mm times 160 mm with a grid size of 21 times 16, which results in 1008 degrees of freedom considering the three model parameters ρ, v_p and v_s. Accordingly, the distance between the grid points is 10 mm. For the time stepping an interval of $1 \cdot 10^{-7}$s is used.

The FWI was performed using the program DENISE from Köhn (2011) and Köhn et al. (2012) and for the OED the genetic algorithm incorporated in MATLAB® was used. Additionally, the x-coordinates of the sources x_S = 70 mm and the receivers x_R = 170 mm were fixed, s. t. they can only be moved on the respective lines and the RMSE(v_s) was calculated for the area of interest as depicted in Figure 1 and used for the objective function. The initial sensor setup for the OED based on the engineer's experience is equally distributing the sources and receivers over the height as depicted in the second row of Figure 2.

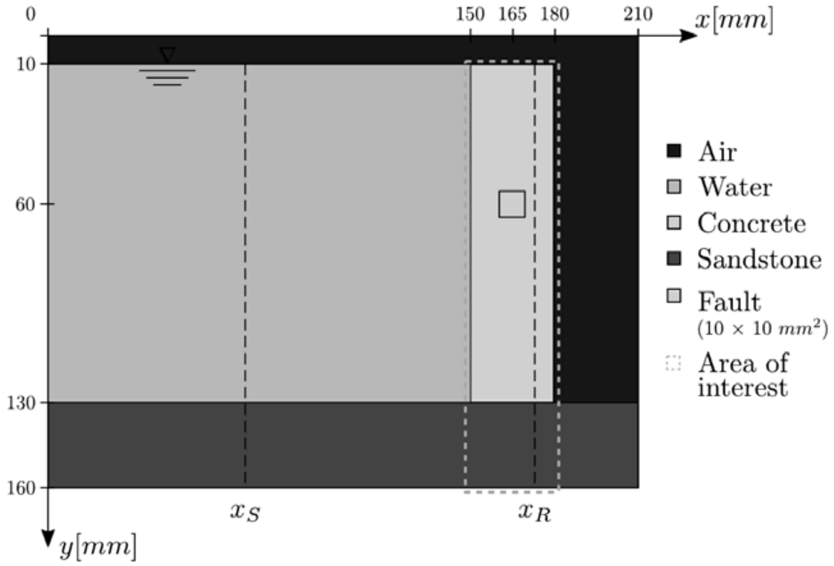

Figure 1. Simplified dam model with fault and area of interest; the dashed lines represent the fixed x-coordinates for the sources and receivers (x_S and x_R, respectively).

4 RESULTS

The results for the distribution of the three model parameters after the FWI are depicted in Figure 2 depending on the positions of the sources and receivers (first column). The first row is showing the true and reference model parameters $\boldsymbol{m}^*$. Additionally, there are three sensor setups with their corresponding values for the RMSE displayed. In the second row the receivers are placed on the top of the dam, in the third row the sources and receivers are distributed equally over the height of the dam's structure and in the last row the gained optimal sensor setup is shown.

5 DISCUSSION

The results of the FWI are very different for the distinct sensor positions as shown in Figure 2. When the receivers are placed on the top of the dam as shown in the second row it leads to a false damage detection in all three model parameters. Here, the fault is not diagnosed as in the reference model, but there is another area at the bottom of the dam where a fault is presumed. This leads to an invalid reconstructed model of the structure, where false prognosis and decision making for possible maintenance is very likely to happen.

Considering the setup in the third row, where the sources and receivers are distributed equally over the height, which is also the initial guess for the experimental design optimization process, there is no damaged area identified for the model parameters density and P-wave velocity. Nevertheless, the evaluation of the S-wave velocity is identifying a fault area exactly at the position of the true fault in the reference model, but still the values of the S-wave velocity are varying a lot from the real values in the dam structure and the soil below.

Regarding the optimal sensor setup in the fourth row that was found by the OED approach, the fault area can be seen in all three model parameters at the exact position, although it is not as clear for the P-wave velocity as for the other two. The results for the optimal design match the real model parameter values best and the damaged area con be identified clearly.

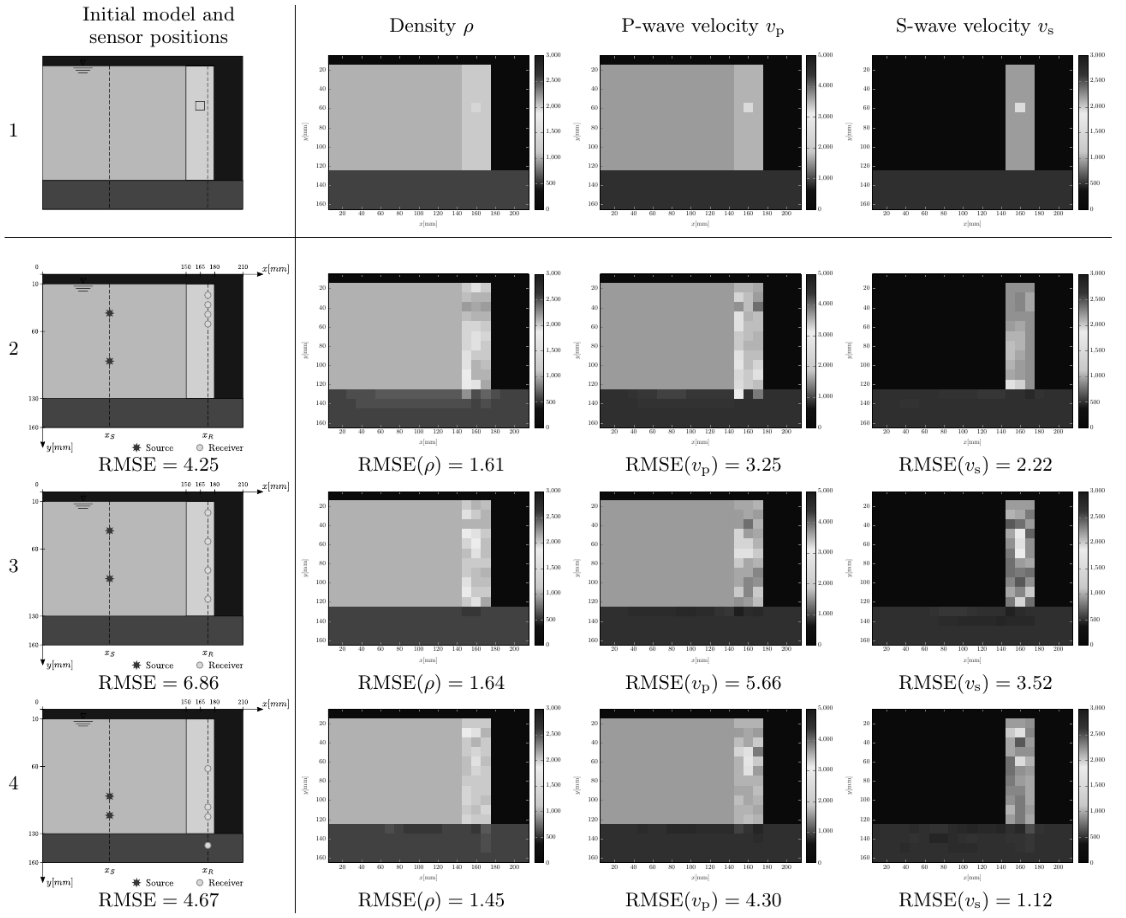

Figure 2. RMSE and FWI results for the three model parameters depending on the initial model and the sensor positions (sources ✸ and receivers ○)

Taking a closer look to the values of the RMSE, unfortunately, a lower RMSE does not always correspond to a better representation of the true model nor a better identification of the fault area. This effect can be seen when comparing the overall RMSE for the top sensor positions and the optimal ones. The overall RMSE for the top positions is less than for the optimal ones, which indicates that putting receivers at the top is superior to the gained optimal positions. Therefore a closer look needs to be taken on the RMSE depending on the model parameters. Here, the RMSE(ρ) and RMSE(v_s) lead to the optimally designed setup, but since the differences in RMSE(v_s) are larger it is here considered for the measure of the objective function.

6 CONCLUSION

The influence of the positions of sources and receivers for the FWI is illustrated in this paper and it is also shown that by optimally planning their placement the quality of the FWI results can be improved. Additionally, the damage detection is possible using optimal sensor positions. On the other hand, non-optimal setups as placing the receivers only on the top of the dam can lead to false damage detection with not-recognizing the real fault area, but misidentifying a damage area where there is no actual damage. This is very problematic since it may lead to a wrong understanding of the structure and in the case of maintenance works they would target a non-problematic area by not considering or repairing the real damage. It is advised to carefully investigate the sensor setup in order to avoid false damage detection. For the mentioned application it was found that the overall RMSE is not very useful as the measure of the objective function, but the RMSE depending only on the S-wave velocity should be considered instead. Similar things account for the visual damage detection when looking at the plots in Figure 2, because the S-wave velocity values display the damaged area best.

Nevertheless, these analyses can be seen as a starting point where many open questions still need to be answered. First of all, the possible sensor positions should be allowed on the whole domain. The findings need to be checked for other positions of the damage and also for multiple ones, too. Also, it is conceivable that optimizing the number of sources and receivers leads to better results. Last but not least, measurement errors need to be introduced, because real experiments are always corrupted by noise.

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