Application of Gaussian process metamodel in structural finite element model updating applying dynamic measured data

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ABSTRACT: Civil infrastructure is vital linking component whose behavior is necessary to be monitored continuously since any fault in performance will cause significant risks. Recently, structural health monitoring (SHM) has obtained a significant contribution in preparing information related to structural behavior during functional life. Though, determining real infrastructure's behavior is intricate, since it relies on structural parameters that cannot be obtained directly from observed data and such identification is prone to uncertainties. Finite element model updating (FEMU) is an approach to address this issue. The current study employs a Modular Bayesian approach (MBA) to update a finite element model (FEM) of a lab-scaled box girder bridge applying natural frequencies. This approach is performed in two stages as undamaged and damaged. These stages can be denoted as the change in structural parameters due to incidences such as impact or fatigue effect. The performed MBA deals with uncertainties thoroughly in all steps. In this study, a discrepancy function is applied to detect the discrepancy in natural frequencies between the FEM and the experimental counterpart. A Gaussian process (GP) is used as a metamodel for the simulated model and the model discrepancy function. In this research, updating the initial FEM of the lab-scale Box Girder Bridge (BGB) by calibrating multi parameters is highlighted. Results specify a considerable drop in stiffness of concrete in damaged phase which is well matched with the cracks observed on the structure's body. Also, discrepancy records reach satisfying range in both stages which implies the structure's properties are predicted accurately.

1 INTRODUCTION

Recently, monitoring structural behavior over their lifespans through information granted by SHM technique has been highly recommended by many researchers such as Frangopol (2011) and Li et al. (2016). One of the qualified approaches to deal with this objective is FEMU which attempts to develop an accurate FEM of real structures. Providing a trustable FEM of structures is efficient in many features such as reliability analysis, assessment of structural performance, damage detection, load carrying capacity assessment (Moravej et al., 2017). One important concern in the field of FEMU is the burdensome computation particularly in case of complex structures which makes this procedure time-consuming and not appropriate in practice (Nishio et al., 2012). While some methods have been presented as computationally efficient such as Response Surface method (Shahidi and Pakzad, 2013) and substructure technique (Weng et al., 2012), it is still essential to address this issue further. Another significant challenge is dealing with different source of uncertainties throughout an updating process of a structural model. So, in most cases, probabilistic methodologies are more realistic than deterministic ones (Jesus et al., 2014; Jesus et al., 2018; Lam et al., 2015; Erdogan et al., 2014). Beck et al. (1998 & 2002) is eminent in utilizing Bayesian approach in SHM. Based on the distinguished study by Kennedy and O'Hagan (2001a), different sources of uncertainty in model prediction have been examined. Higdon et al. (2008) applied a comprehensive MBA, developed earlier by Kennedy and O'Hagan (2001a), but was not generally effective by reason of lack of identifiability (Arendt et al., 2012a; Yuen, 2010). Arendt et al. (2012b) offered a development to the Kennedy and O'Hagan original formulation to conquer the identifiability issue, through using measured data with various responses. In this technique, FEM is swapped with a GP metamodel, which considerably decreases the computational work (Lophaven et al., 2002; Wan et al., 2014; Spiridonakos et al., 2015; Jesus et al., 2017). Consequently, this formulation is comprehensive enough to consider existing uncertainties and superior to the previous attempts in model updating due to comprising all types of uncertainty and accordingly obtain the more trustworthy result. Based on such a significant efficiency of MBA, this study investigates its practical performance in FEMU with use of measured vibration data to tune multiple parameters whereas most of the former studies in MBA applied only single parameter. The goal of the study is to examine the applicability of the algorithm on a lab-scaled reinforced concrete BGB, a common operating bridge in Australia, in two stages as undamaged and damaged settings. The damaged phase in this study, generated by applying a point load and a cyclic load, is a representation of changes in structural parameters as a result of significant incidences such as impact and fatigue, and this study intends to detect these changes and deliver an updated model at each step which can be considered as a tool to identify the capacity of structure in each stage and observe its performance. This test is the first practical application of this approach in model updating in two settings as undamaged and damaged. Besides, in this study, multiple parameters are calibrated while the previous studies mostly focused on a single parameter

2 METHODOLOGY

The comprehensive equation of model updating is presented in Eq (1) which indicates the output of a numerical model within a domain of calibrated parameters θ = θ* has an appropriate fit with the experimental data.

$$Y_e = Y_m\left(\theta^*\right) + \delta + \varepsilon \qquad (1)$$

Where Y_e is the observed responses, ε_T= [ε1, …, εn] is the observation error vector, $Y_m(\theta^*)$ is the model output and θ* are a r-dimensional vector of structural parameters, and δ is the discrepancy function that interprets the difference between the model and the true process. In this research, the discrepancy function and the numerical model will be replaced with multiple response Gaussian process MRGP. Each MRGP has specific hyperparameters that define it and they should be estimated.

2.1 Gaussian Process

GP modeling is a method for interpolation with addressing uncertainty (Kennedy and O'Hagan., 2001a; Sacks et al., 1989; Rasmussen et al., 2006; Jin et al., 2001). To produce this GP, it is required to obtain the mean function at every design input point. If the measured data is located outside or between the design input points, MRGP must find a possible extrapolation or interpolation from the existing data. In MRGP, the prior mean function is considered as a member of a hierarchical structure of linear functions in a generalized form of M = Hβ. Herein, matrix H comprises N polynomial constant regression functions and the matrix of regression coefficient β, for each term included in matrix H and each fitted response in Y. In the other word, H is a row vector of regression functions and β is a column vector of regression functions. The prior covariance function of MRGP which defines the model and discrepancy function can be formulated as Eq (2).

$$V = \Sigma^2 R \tag{2}$$

Where $\Sigma^2 \in \mathbb{R}^{q \times q}$ is a spatial variance matrix, $R \in \mathbb{R}^{N \times N}$ is a correlation matrix. The matrix R includes a correlation function for each of its entries which must be approximated. In the current work, the model has been expected to follow a linear function as mentioned in (Lophaven et al., 2002), because it fits better to the data and is numerically more stable. Each GP modeling is identified by its specific hyperparameters. After providing a certain amount of data Y to the MRGP, the posterior distribution of the response should be obtained and the hyperparameters must be estimated. As a reason of providing better computational efficiency, the hyperparameters are calculated with the maximum likelihood estimates (MLEs) in this study.

2.2 Modular Bayesian approach (MBA)

The MBA divides the progression of updating into four modules, and hyperparameters of the MRGP are estimated apart each other and consecutively (Kennedy and O'Hagan, 2001b). As it is shown in the study by Arendt et al., (2012a), at each stage, the hyperparameters of metamodels are estimated and fixed. This act of estimating and fixing the hyperparameters is carried out successively when passing on from module 1 to module 2 and from module 2 to module 3. Estimation is directed through numerical optimization methods by matching the likelihood among the MRGP and the obtained data. In the current work, a MATLAB genetic algorithm (GA) routine was used to estimate the parameters of a GP which approximates the discrepancy function. This is carried out by maximum likelihood estimation (MLE), which indicates that the fitness function of the GA is the likelihood function. In module 3, Bayes' theorem is applied to approximate the posterior distribution of parameters and its likelihood function contains the two MRGP approximated in modules 1 and 2. This research applied MBA for multiple calibration parameters with using a Markov Chain Monte Carlo method.

3 FINITE ELEMENT MODEL UPDATING OF THE STRUCTURE

A six meter long reinforced concrete BGB, built in Queensland University of Technology (QUT) laboratory, was considered in this study, the structure was cast in three parts subsequently as top slab, web and bottom slab. The BGB's dimensions are illustrated in Fig 1. More details about casting of the structure can be found in (Pathirage, 2017).

Figure 1. The BGB dimensions

The BGB was relocated from its initial place which was on the ground, onto two simple supports as pin and roller. This platform denotes the first stage (undamaged stage), despite minor cracks existed beneath the soffit slab the relocation. In the second stage (damaged stage), a point load and a cyclic load were applied at the mid-span of the BGB which lead to some significant cracks under the soffit slab and the web of the BGB.

3.1 Numerical model

In this study, nominal values of the parameters from the design step were used, since the information about the current state of the structure is not available. The first BGB's finite element model was constructed in ABAQUS 2017 software package as shown in Fig. 2.

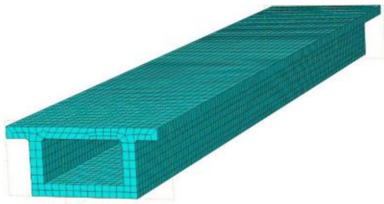

Figure 2. FE model of BGB built in ABAQUS

In the simulated model, C3D8R solid element and T3D2 truss element were selected for concrete and reinforcement elements, respectively. The initial material properties based on available design information are demonstrated in Table 1. Furthermore, for both supports, the boundary condition was assigned as fixed in vertical translation.

Table 1. Material properties used in FE model of BGB

Parameter	Material	Nominal values as designed
Young Modulus E (GPa)	Concrete	32
	Reinforcement	200
Mass Density ρ (Kg/m3)	Concrete	2400
	Reinforcement	7850

In this work, four natural frequencies of the FE model as the 1st vertical bending, 2nd vertical bending, 1st lateral bending, and 3rd vertical bending modes are considered for the updating process.

3.2 Modal data measurement

The vibration responses of the structure in both undamaged and damaged stages were measured by accelerometers and used in the FEMU process. To select the correct position for attaching the sensors, various aspects were considered with respect to the excitation source, maximum modal displacement points, and available type and number of sensors. Captured acceleration responses were applied in a modal analysis practice, Stochastic Subspace Identification (SSI) method embedded in the ARTeMIS Modal software package. The four natural frequencies as the 1st vertical bending, 2nd vertical bending, 1st lateral bending and 3rd vertical bending modes in order were used for the updating process since they were detectable in both stages. The natural frequencies in first numerical model and experimental in both stages are shown in Table. 2.

Table 2. Frequency in the initially designed model and measured data in two stages

Mode No	Freq as designed (Hz)	Freq measured in Undamaged case (Hz) (Mean value)	Error (%)	Freq measured in Damaged case (Hz) (Mean value)	Error (%)
1	24.339	21.65	-12.42	18.78	-29.60
2	81.29	67.06	-21.22	63.06	-28.9
3	92.108	84.32	-9.24	80.73	-14.09
4	109.75	98.21	-11.75	95.74	-14.6

In this study, a sensitivity analysis was run for selecting appropriate parameters which are sensitive enough to the four selected natural frequencies by using FEMtools software package (DDS, 2012). After selecting various types of parameters, the parameter selection was ended up with five most sensitive parameters to the responses as Young's modulus values of the concrete bottom slab, web and top slab (EcBot, EcWeb & Ectop) and vertical Spring Stiffness of both simply supports (Kroller & Kpin).

4 RESULT AND DISCUSSION

The FE model of the BGB was calibrated in two stages by using MBA. In this study, the prior PDF for the parameters are selected according to previous studies and codes of practice (Darmawan and Stewart, 2007; Mirza et al., 1979; Mirza et al., 1980) and (AS 5104). So the Young's modulus of concrete for all parts was considered as a normal distribution with a Mean value of 32 GPa, Coefficient of Variation equals 7.13, and for both boundary conditions, vertical spring stiffness was considered as a normal distribution with mean of 5*107 N/m and Coefficient of Variation equals 9*1013. After obtaining the hyperparameters of MRGP to approximate numerical model and the discrepancy function in module 1 and 2, which represents the GPs, the results for calibrated parameters after applying MBA in the undamaged stage are obtained in module 3, as illustrated in Fig. 3.

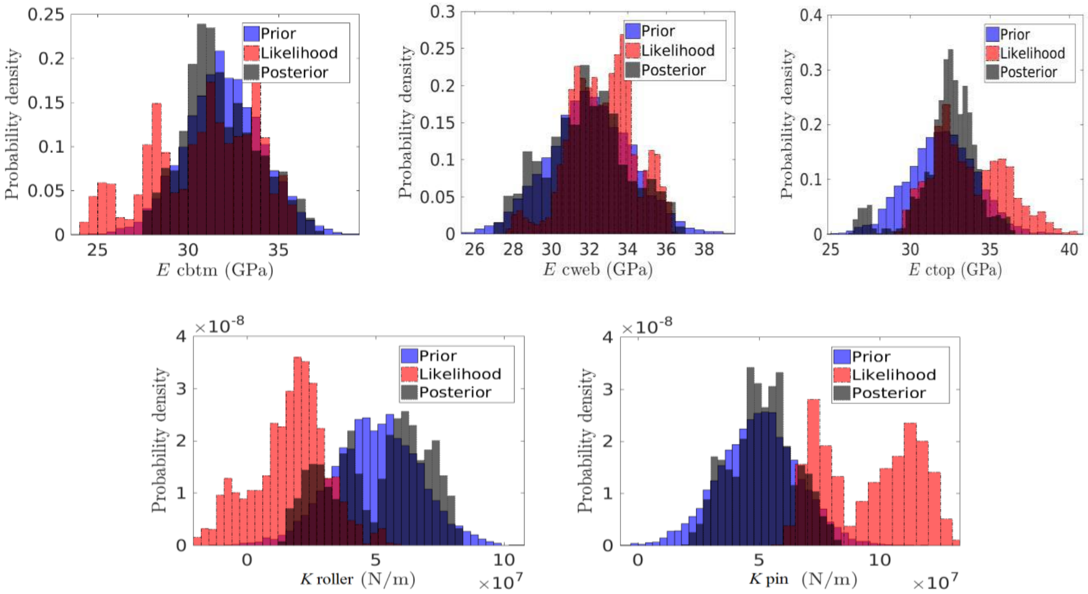

Figure 3. Prior, Max Likelihood and Posterior PDF for calibrated parameters in undamaged stage

In the current work, the likelihood determines the updated parameters according to the measured data. The posterior may need more data to represent the calibrated parameters reliably. As can be seen in Fig. 3, there are no significant changes in likelihood against with priors in Young's moduli of the web and the top slab. The considerable change can be noted as the decrease in the bottom slab's Young's modulus which is matched with the observed minor cracks underneath the BGB. The most noticeable change would be the drop in vertical spring stiffness at roller support which represents the overestimation of vertical fixity of roller support before updating. It is found that the discrepancy function in the undamaged stage for four natural frequencies reaches the responses with a deviation of less than 6% compared to the measured ones. In the next step, the damaged structure is updated. The parameters of Young modulus of all three parts reduced in the damaged stage. The calibrated parameters distribution as Prior, Likelihood and Posterior are illustrated in Fig. 4.

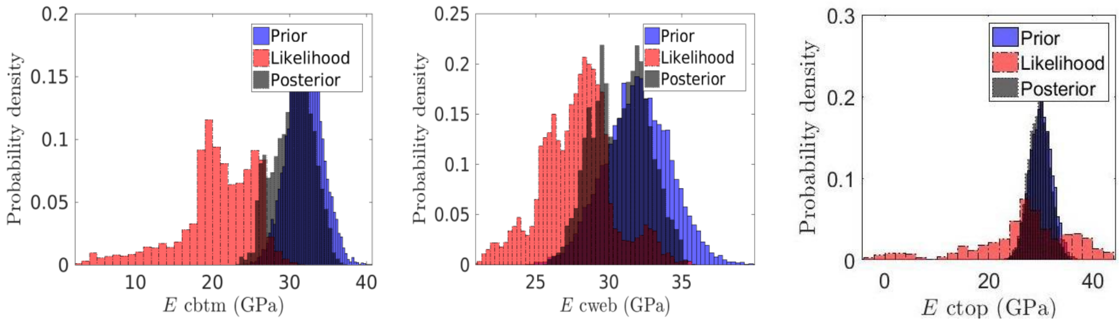

Figure 4. Prior, Max Likelihood and Posterior PDF for calibrated parameters in damaged stage

As it is illustrated in this figure, the substantial change is targeted at the likelihood in Young's modulus for the bottom slab which reduces to 20 GPa, representing a drop of 37%. In addition, the reduction in Young's modulus in the web is noticeable showing 27 GPa of its likelihood mean. Meanwhile, the top slab was not much affected, and its Young's modulus is very close to its prior. The reduction in Young's moduli of the bottom slab and the web are well matched with the cracks observed in the damaged stage. It is found that the discrepancy in damaged stage arised, particularly for the 2nd and 3rd vertical bending modes. It could be described as the cracks may produce nonlinearities in structural materials as well as in structure's response mechanism.

5 CONCLUSIONS

In this study, performance of the developed MBA on a BGB was examined by means of vibration data. MRGP as a metamodel was used to approximate the whole numerical model and it expedited the computational process more than other probabilistic updating techniques. This advantage makes this method remarkable, especially in case of complicated structures. Furthermore, it is the first time in applying MBA on two stages as damaged and undamaged which can be represented as a structure's state during its life period and the result could be applied to reliability analysis, performance monitoring, and damage severity estimation. On the contrary to most of the previous studies which applied MBA for a single parameter, this study is an extreme example of model updating with five parameters at the same time, and changes of these parameters were well fitted with the observed evidence in both stages. The model has been updated and the updated model was adequately matched with physics of damaged beam. Also, the results in the damaged stage illustrated, the rise of discrepancy function. This observation can be concluded as a significant point for designers and indicates that the FE model requires to be refined by considering more aspects such as modelling cracks with the change of cross section area. Although, modal frequencies has been applied to calibrate the model in this study, other types of responses such as mode shapes can be used in the future works. Furthermore, the approach has the capability to consider various ambient variables as temperature, humidity, and wind speed to decrease the degree of uncertainty in model calibration and in turn reach the more reliable outcome.

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