Design Optimisation of Cable-Stayed Bridge Based on Cable-Bridge Resonance Control

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ABSTRACT: Multi-cable systems are usually applied to modern cable-stayed bridges. In such bridges, some steel cables are prone to unfavourable cable-bridge resonance, if the whole bridge's vibration frequencies match the natural frequencies of these cables. To avoid this problem, optimisation based on cable-bridge coupled vibration control should be account in design. In this paper, such optimisation process for a real cable-stayed bridge is presented. A fine finite element model of the cable-stayed bridge was established and the non-linear dynamic analysis was performed. Then the cable material of the stay cables that occur resonance were changed from steel to CFRP. CFRP (short for carbon fibre reinforced polymer) is an advanced composite material, whose strength is higher than steel but density is much lower. The results show that changing the cable material from steel to CFRP and hence increasing the cable natural frequency can effectively prevent the occurrence of cable-bridge resonance. This paper can provide a useful reference to bridge designers.

KEYWORDS: Cable-stayed bridge; cable-bridge resonance; optimisation; dynamic analysis

1 INTRODUCTION

Reviewing the history of cable bridges, it can be found that the development of cable materials can significantly promote the development of bridges. Carbon Fibre Reinforced Polymer (CFRP) is an advanced composite material with advantages of high strength, lightweight, no corrosion and high fatigue resistance, which makes it suitable to be made into cables and replace steel cables in cable bridges and eventually promote their development (Yue 2015).

Back in the 1980s, several experts have considered using CFRP cables in cable bridges. In 1987, Meier proposed the conception of building a CFRP cable-stayed bridge with a main span of 8400 m crossing the Strait of Gibraltar. From then on, more and more CFRP cable bridges have been studied, designed and even constructed (Meier 2012).

However, the existing researches usually only consider the static advantages of the CFRP cables, and the potential dynamic advantages of the CFRP cables are still lack of research. This paper proposes a new application of CFRP cables, that is, using them to control the cable-bridge coupled vibration of cable-stayed bridges. First, the theory of cable-bridge coupled vibration is demonstrated. Then, a case study is presented. A fine finite element model of a cable-stayed bridge was established and the non-linear dynamic analysis was performed. The stay cables that occur resonance were redesigned through changing the cable material from steel to CFRP. The results show that the this can significantly reduce the cable-bridge resonance.

2   THEORETICAL BASIS: STAY CABLE VIBRATION

2.1   *Stay cable vibration model*

The model of a typical stay cable and its boundary conditions can be shown in the Figure 1. This cable is excited not only at its end on the bridge deck by the excitation $X_d sin\Omega t$, but also at the end on the bridge pylon by the excitation $X_p sin\Omega t$. Because the present cable stay-bridges usually apply multi-cable system, the deck's excitation and pylon's excitation have the same frequency. But their amplitudes are usually different.
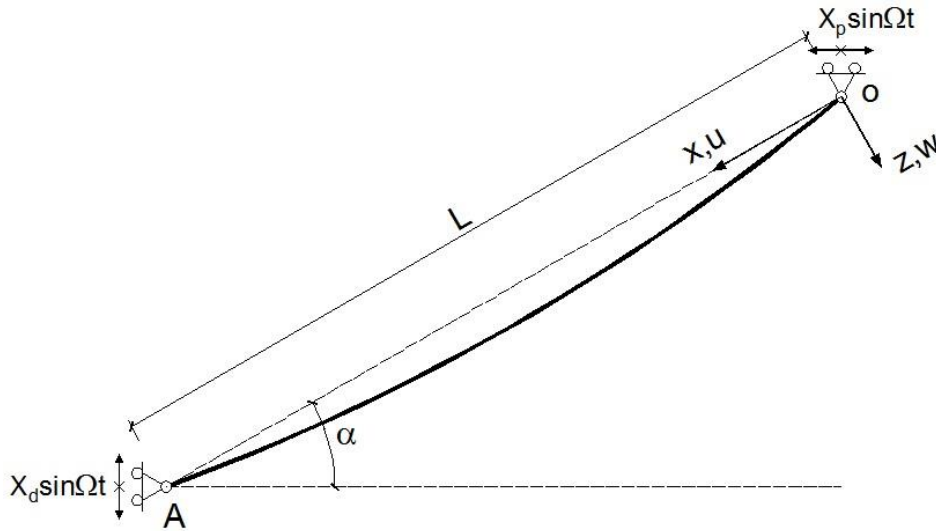

Figure 1: Stay cable vibration model

Take a micro element out of the cable arbitrarily. Its dynamic equilibrium is shown in the Figure 2.
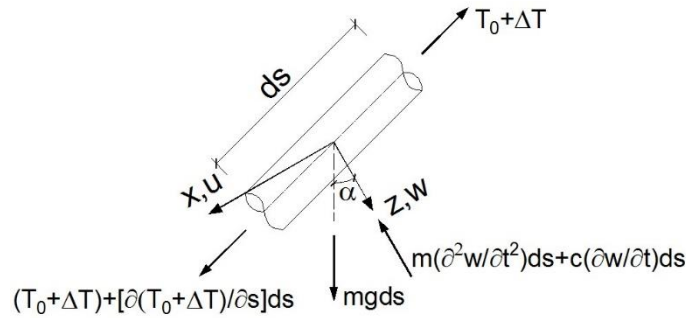

Figure 2: An arbitrary micro element from the cable in the dynamic equilibrium

Consider the dynamic equilibrium in the Z direction, scilicet $\sum Z = 0$:

$$(T_0 + \Delta T)\frac{\partial^2 w}{\partial x^2} + \Delta T \frac{\partial^2 z}{\partial x^2} = m\frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} \tag{1}$$

Where the *ds* indicates the element length, and the *s* is arc length coordinate; the T_0 indicates the static tension force in the cable, and the ΔT represents the dynamic tension force, which is the function of time *t*; the *mg* indicates the cable gravity of the unit length; *m* and *c* are the mass of unit cable length and the damp of unit cable length, respectively.

Furthermore, the dynamic tension force ΔT can be represented as:

$$\Delta T = EA\varepsilon(t) \tag{2}$$

Where E is the E-Modulus and A is the cross-section area of the cable; $\varepsilon(t)$ indicates the dynamic strain, which is the average strain along the cable and is the function of the time t.

Substitute (2) into (1) and apply the Galerkin Method (Cockburn et al. 2011), one can obtain:

$$w(x,t) = \left[(X_d \cos\alpha + X_p \sin\alpha)\frac{x}{L} - X_p \sin\alpha \right]\sin\Omega t + w(t)\sin\frac{\pi x}{L} \tag{3}$$

Then, according to the D'Alembert's principle (Vujanovic 1978), the vibration formula of the stay cable which is excited at both deck and pylon ends can be established as follows:

$$\ddot{w}(t) + 2\xi\omega_0\dot{w}(t) + \omega_0^2\left(1 + \frac{1}{2}\left(\frac{2}{\pi}\right)^4\lambda^2 + \frac{B}{X_0}\sin\Omega t + \frac{D^2}{2LX_0}\sin^2\Omega t\right)w(t) + \frac{3\pi T_0\chi}{mLX_0}w^2(t)$$

$$+ \frac{\pi^4 T_0}{4mL^3 X_0}w^3(t) =$$

$$-\frac{2\Omega^2}{\pi}(2X_p\sin\alpha + D)\sin\Omega t + 2\xi\omega_0\frac{2\Omega}{\pi}(2X_p\sin\alpha + D)\cos\Omega t - \frac{4T_0\chi}{\pi mX_0}\left(B + \frac{D^2}{2L}\sin\Omega t\right)\sin\Omega t \tag{4}$$

Where

$$\frac{mg\cos\alpha}{T_0} = \chi \quad \text{(Curvature)} \tag{5}$$

$$X_d\sin\alpha - X_p\cos\alpha = B \tag{6}$$

$$X_d\cos\alpha + X_p\sin\alpha = D \tag{7}$$

$$\omega_0 = \frac{\pi}{L}\sqrt{\frac{T_0}{m}} \quad \text{(Natural frequency)} \tag{8}$$

$$\xi = \frac{c}{2m\omega_0} \quad \text{(Damping ratio)} \tag{9}$$

$$\lambda^2 = \left(\frac{mgL\cos\alpha}{T_0}\right)^2\frac{EAL}{L_e T_0} = \frac{EA}{L_e}\frac{L^3\chi^2}{T_0} \tag{10}$$

$$X_0 = \frac{T_0 L_e}{EA} \quad \text{(The elastic extension ratio of the stay cable)} \tag{11}$$

2.2 Solution of stay cable vibration formula

The vibration formula of the stay cable (Equation (4)) is very difficult to get the analytical solution, but relatively easy to obtain the numerical solution. Take a steel stay cable from the Guadiana Bridge as an example, its properties are listed in Table 1.

Table 1

L (m)	α (°)	m	EA	T_0	χ	X_0 (m)	λ^2	ω_0 (Hz)

		(kg/m)	(kN)	(kN)				
49.485	67.8	21	518700	2141.7	3.62×10^{-5}	0.2043	7.77×10^{-4}	3.2268

Assume the $\xi = 0$, the $X_d = 10cm$ and $X_p = 5cm$, change the excitation frequency Ω, and solve the Equation (4) by the numerical method. Then, the stay cable's amplitude-frequency curve can be obtained.
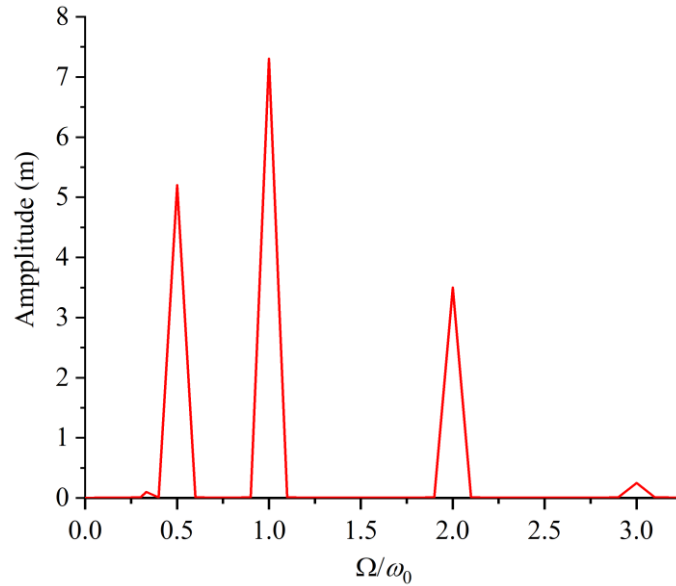

Figure 3: Amplitude-frequency of stay cable under external excitation

As can be seen from Figure 3, when the ratio of the excitation frequency to the natural frequency of the stay cable (Ω/ω_0) equals to 1/3, 1/2, 1, 2 or 3, amplitude peaks can be observed. This means when Ω/ω_0 = 1/3, 1/2, 1, 2 or 3, especially Ω/ω_0 = 1, the cable-bridge resonance will happen. If the bridge vibration frequency Ω is fixed, changing the cable's natural frequency ω_0 can be an effective way to avoid the potential cable-bridge resonance.

3   CASE STUDY: USING CFRP CABLES TO AVOID CABLE-BRIDGE RESONANCE

3.1   *Natural frequency comparison between steel and CFRP stay cables*

The density of CFRP is approximately 1/5 that of steel. Moreover, CFRP cables do not need strict anticorrosion measures like steel cables. This makes the line weight *m* of CFRP cable considerably smaller than that of steel cable. Furthermore, according to Equation (8), the cable's natural frequency ω_0 is inversely proportional to the square root of *m*, which indicates that changing steel to CFRP will significantly increase the ω_0.

For example, changing the steel stay cable from the Guadiana Bridge (see Table 1) to the CFRP cable will make its *m* decrease to 2.625 kg/m. Assume other conditions remain the same, using Equation (8) the natural frequency of the CFRP cable can be calculated to 9.1266 Hz, which is approximately 2.83 times as the steel cable's natural frequency (3.2268 Hz).

This comparison infers that replacing steel cables with CFRP cables can avoid the situations of Ω/ω_0 = 1/3, 1/2, 1, 2 and 3 and thus avoiding the cable-bridge resonance.

3.2 *Replacing steel cables with CFRP cables in an example cable-stayed bridge*

To prove the aforementioned inference, an example cable-stayed bridge was analysed through the FEM in the software SOFiSTiK (SOFiSTiK 2018). The investigated cable-stayed bridge is a foot bridge with single cable plane. Its geometry is shown in Figure 4.
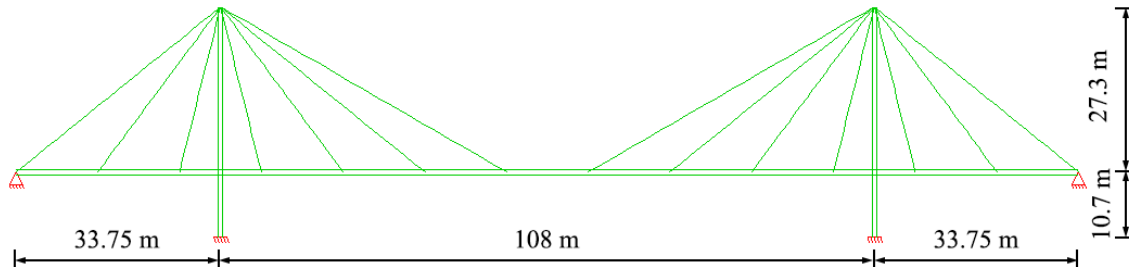

Figure 4: Geometry of investigated cable-stayed bridge

First, the cable-stayed bridge was designed through static calculation. Its reasonable bridge completion state was found (see Figure 5) and the cross-sections of cables, pylons and girder were designed.
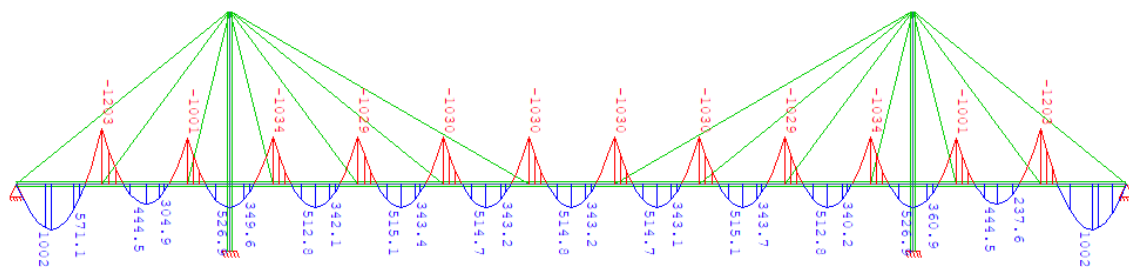

Figure 5: Bending moment diagram of the bridge completion state (unit: kN·m)

Then, the non-lineal dynamic analysis was conducted and the stay-cables which were prone to the cable-bridge resonance were found out. For example, when the bridge second order mode vibration happened (frequency = 1.71 Hz), the two stay cables in the mid-span occurred the obvious cable-bridge resonance (see Figure 6), because their natural frequency (= 1.56 Hz) is close to the bridge vibration frequency.
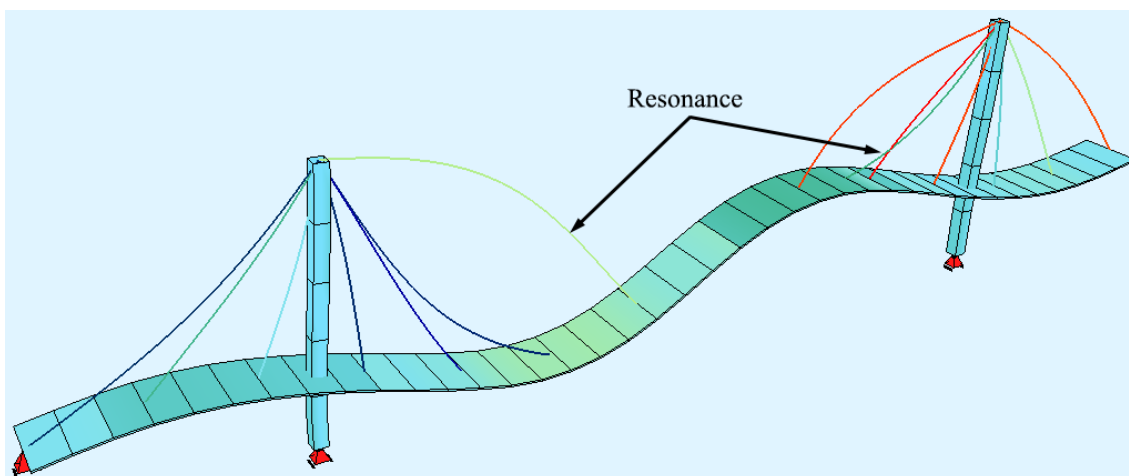

Figure 6: Obvious cable-bridge resonance happening at the two stay cables in mid-span

Change the resonance steel cables to CFRP cables and redo the non-lineal dynamic analysis, one can obtain the vibration diagram as Figure 7.
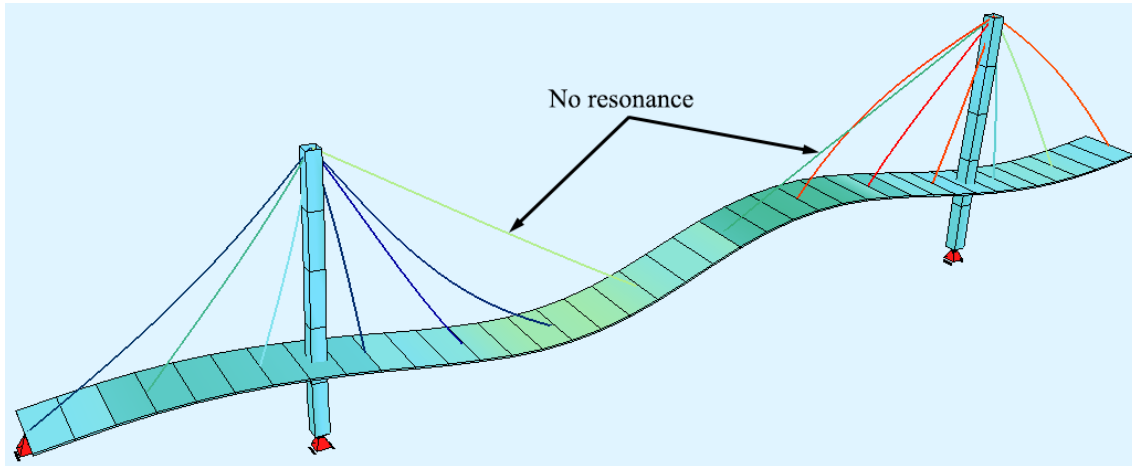

Figure 7: No cable-bridge resonance happening at the two stay cables in mid-span

As can be seen from Figure 7, replacing the resonance steel cables with the CFRP cables avoids the cable-bridge resonance, because the natural frequency of the stay cables was increased from 1.56 Hz to 3.74 Hz and thus the situations of $\Omega/\omega_0 = 1/3, 1/2, 1, 2$ and 3 were successfully avoided.

4   CONCLUSION

In this paper, the optimisation based on cable-bridge coupled vibration control using CFRP stay cables is presented. Theoretical analysis was conducted and the situations prone to cable-bridge resonance were found out. A case study on an example cable-stayed bridge was performed and its steel resonance stay cables were changed to CFRP cables. The key conclusions drawn from this work are:

(1)   When the ratio of the bridge vibration frequency to the natural frequency of the stay cable (Ω/ω_0) equals to 1/3, 1/2, 1, 2 or 3, especially $\Omega/\omega_0 = 1$, the cable-bridge resonance will occur.

(2)   The bridge vibration frequency Ω is hard to change, and an easier way to avoid the cable-bridge resonance is to change the cable's natural frequency ω_0.

(3)   Replacing the resonance steel cables with the CFRP cables will significantly increase the cable natural frequencies and avoid the potential cable-bridge resonance.

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