# Estimating seismic interstory drifts of building structures using time-varying shear model with acceleration data

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ABSTRACT: Structural health monitoring (SHM) of important building structures, used as disaster-response bases, financial centers, hospitals and so on, in earthquake-prone areas are essential because it can facilitate rapid decision making on evacuation and re-occupancy after a great earthquake. Interstory drift is an important engineering demand parameter and key indicator of structural performance owing to its correlation best with seismic damage of building structures. This paper presents a time-varying shear model-based method for estimating seismic drift responses of building structures using acceleration measurements at a limited number of stories. A drift estimation algorithm is first formulated on the basis of state estimation of a time-varying shear building model using the unscented Kalman filter. The parameters of stiffness and damping are time-variant considering the fact that structural stiffness and damping vary during earthquakes. Then, the effectiveness of the presented method is numerically investigated through a four-story hysteretic shear building model subject to an earthquake motion recorded in the 1989 Loma Prieta earthquake.

## 1 INTRODUCTION

Maximum interstory drift is a key indicator for structural seismic performance evaluation owing to its correlation best with seismic damage of structures. In performance-based earthquake engineering methods, knowing interstory drifts allows for damage evaluation of buildings by means of the fragility functions which map maximum interstory drift ratio to the level of seismic damage of structural and nonstructural components (Ozturk (2003; 2006)). In addition, limits on interstory drift ratios are usually used to ensure structural deformation at acceptable levels in seismic design or rehabilitation. For example, interstory drift limits are utilized to determine building performance levels (e.g., immediate occupancy, life safety, and collapse prevention) in FEMA-356. Thus, accurately estimating interstory drifts would facilitate performance evaluation and damage assessment of building structures in an objective manner following major earthquakes.

Structural health monitoring (SHM) of important building structures, used as disaster-response bases, financial centers, hospitals and so on, in earthquake-prone areas are essential because it can facilitate rapid decision making on evacuation and re-occupancy after a great earthquake (Li et al. (2015; 2016; 2017; 2018)). In recent years, a few building structures located in seismically active regions have been instrumented with accelerometers under strong-motion instrumentation programs (SMIP), such as the California SMIP of California Geological Survey. Given the availability of acceleration data from the strong motion monitoring systems, estimation of displacements/drifts from direct signal processing of measured acceleration data, including numerical double integration combined with interpolation, are commonly used for damage assessment and safety evaluation of building structures (Naeim et al. (2006)). For example, Çelebi et al. (2004) reported a real-time double integration algorithm for computing lateral displacements and drift ratios from signals of 30 accelerometers deployed on a 24-story building in San Francisco, CA. Nonetheless, this method of estimating displacement response is quite sensitive to measurement noise and requires high-quality measurements of acceleration responses. Furthermore, double integration of acceleration is unreliable and questionable for estimating nonlinear displacements (Skolnik et al. (2010)).

This paper presents a time-varying shear model-based method for estimating seismic drift responses of building structures using a limited number of acceleration measurements. Drift estimation algorithm is formulated on the basis of state estimation of a time-varying shear building model using the unscented Kalman filter. In the method, the parameters of stiffness and damping are treated to be time-variant considering the fact that structural stiffness and damping vary during earthquakes in terms of the system identification results from seismic response data. The effectiveness of the presented method is numerically investigated through a four-story hysteretic shear building model subject to an earthquake motion recorded in the 1989 Loma Prieta earthquake.

## 2 DRIFT ESTIMATION METHOD

### *2.1 Time-varying shear model*

A time varying shear building model is used to simulate structural dynamics of a building subjected to seismic ground motion. The equation of motion for the structure is given by

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}(t)\dot{\mathbf{x}}(t) + \mathbf{K}(t)\mathbf{x}(t) = -\mathbf{M}\mathbf{1}\ddot{x}_g(t) \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is relative displacement vector at time $t$; $n$ is the number of degrees of freedom; $\ddot{x}_g(t)$ denotes seismic ground acceleration; $\mathbf{1} \in \mathbb{R}^n$ is a vector with each element equal to unity; $\mathbf{M} \in \mathbb{R}^{n\times n}$ denotes mass matrix, which is assumed to remain constant during the earthquake and calculated according to the structural drawings; $\mathbf{C}(t) \in \mathbb{R}^{n\times n}$ and $\mathbf{K}(t) \in \mathbb{R}^{n\times n}$ are time varying damping matrix and time varying stiffness matrix.

During a damaging earthquake, buildings might sustain seismic damage, such as cracking of concrete slabs, and local buckling and fracture of steel beams, which impairs interstory stiffness of buildings. Thus, we suppose that the interstory stiffness of buildings are time-invariant and the stiffness matrix is defined as

$$\mathbf{K}(t) = \sum_{i=1}^{n}[1-\delta_i(t)]k_i\mathbf{K}_i \tag{2}$$

where $\mathbf{k} = [k_1, k_2, \ldots, k_n]^{\mathrm{T}}$ denotes the interstory stiffness vector of the undamaged structure before earthquakes, which are identified from acceleration measurements with structural model updating methods; $\boldsymbol{\delta} = [\delta_1(t), \delta_2(t), \ldots, \delta_n(t)]^{\mathrm{T}}$ is a time-varying reduction factor vector associated with the interstory stiffness vector $\mathbf{k}$; and $\mathbf{K}_i \in \mathbb{R}^{n\times n}$ ($i = 1, 2, \ldots, n$) are the substructure stiffness matrices, which are determined by structural analysis such as the matrix displacement method. Unlike most literature where the damping is assumed to be known and time-invariant (Shan et al. (2015)), the damping matrix is treated as Rayleigh damping with a time-variant coefficient considering the fact that modal damping ratios vary during earthquakes in terms of the damping identification results from seismic response data (Huang et al. (2004)) as follows

$$\mathbf{C}(t) = a(t)\mathbf{M} + b\mathbf{K}(t) \tag{3}$$

where $a(t)$ is a time-variant coefficient; because the stiffness matrix $\mathbf{K}(t)$ is time-variant the coefficient $b$ is assumed to be constant and it is defined as

$$b = \frac{2\zeta}{\omega_1 + \omega_2} \tag{4}$$

where parameter $\zeta$ is set to be 0.05 which corresponds to the initial damping ratio of 5%; $\omega_1$ and $\omega_2$ are the first two natural frequencies of the structure before earthquakes.

By defining the state vector $\mathbf{z}(t) = [\mathbf{x}^T \dot{\mathbf{x}}^T a \boldsymbol{\delta}^T]^T$, Eq. (1) can be expressed in first-order state space form as

$$\dot{\mathbf{z}}(t) = \frac{d}{dt}\begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \\ a \\ \boldsymbol{\delta} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{x}} \\ -\mathbf{M}^{-1}(a\mathbf{M} + b\mathbf{K}_o)\dot{\mathbf{x}} - \sum_{i=1}^{n}(1-\delta_i)k_i\mathbf{M}^{-1}\mathbf{K}_i\mathbf{x} \\ 0 \\ \mathbf{0}_{n\times1} \end{bmatrix} - \begin{bmatrix} \mathbf{0}_{n\times1} \\ \mathbf{1}_{n\times1} \\ 0 \\ \mathbf{0}_{n\times1} \end{bmatrix}\ddot{x}_g + \begin{bmatrix} \mathbf{0}_{n\times1} \\ \mathbf{0}_{n\times1} \\ 1 \\ \mathbf{1}_{n\times1} \end{bmatrix}\gamma(t) \tag{5}$$

where $\mathbf{0}_{i\times j}$ = $i\times j$ matrix whose elements are zeros; $\mathbf{1}_{i\times j}$ = $i\times j$ matrix whose elements are ones; the coefficient $a(t)$ and reduction factor $\delta_i(t)$ ($i$ = 1, 2, …, $n$) are assumed to be random walks with time, i.e., $\dot{a} = \gamma(t)$ and $\dot{\delta}_i = \gamma(t)$, where $\gamma(t)$ is an independent Gaussian noise. The process noise $\mathbf{w}(t) \in \mathbb{R}^{2n+1}$ is defined as

$$\mathbf{w}(t) = \begin{bmatrix} \mathbf{0}_{2n\times1} \\ \mathbf{1}_{(n+1)\times1} \end{bmatrix}\gamma(t) \tag{6}$$

Suppose that absolute acceleration at $m$ ($\leq n$) stories of the structure are measured and the acceleration measurements $\mathbf{y}(t) \in \mathbb{R}^m$ are modeled as

$$\mathbf{y}(t) = \boldsymbol{\Gamma}\left[-\mathbf{M}^{-1}(a\mathbf{M} + b\mathbf{K}_o)\dot{\mathbf{x}} - \sum_{i=1}^{n}(1-\delta_i)k_i\mathbf{M}^{-1}\mathbf{K}_i\mathbf{x}\right] + \mathbf{v}(t) \tag{7}$$

where $\boldsymbol{\Gamma}$ is an $m \times n$ selection matrix with the elements of 1 or 0, which picks the components associated with the measured acceleration data; $\mathbf{v}(t) \in \mathbb{R}^m$ is the measurement noise, which is modeled as an independent Gaussian noise. Eqs. (5) and (7) can be written more compactly in state space as

$$\dot{\mathbf{z}}(t) = f(\mathbf{z}(t), \mathbf{w}(t)) \tag{8}$$

$$\mathbf{y}(t) = h(\mathbf{z}(t), \mathbf{v}(t)) \tag{9}$$

where $f(\cdot)$ is the state transition function; $h(\cdot)$ is the observation function.

### *2.2 Kalman filtering*

In this study, the augmented unscented Kalman filter (UKF) (Wu et al. (2007)) is employed in state estimation for estimating the state $\mathbf{z}(t)$ of the system, in which the state is augmented with the process and measurement noises. Eqs. (8) and (9) are rewritten as discrete-time state-space representation to implement the augmented UKF as follows

$$\mathbf{z}_{k+1} = f_d(\mathbf{z}_k, \mathbf{w}_k) \tag{10}$$

$$\mathbf{y}_k = h(\mathbf{z}_k, \mathbf{v}_k) \tag{11}$$

where $\mathbf{z}_k$ is the state vector at $t_k = k\Delta t$ ($k = 1, \ldots, N$, $N$ is total number of measurements; $\Delta t$ = discretization time step); $\mathbf{y}_k$ is the measurement vector at time $t_k$; $\mathbf{w}_k$ is the discrete process noise at time $t_k$ and it is assumed to be a Gaussian vector with zero mean and covariance matrix $\mathbf{Q}$; $\mathbf{v}_k$ is the measurement noise at time $t_k$, which is assumed to be a Gaussian vector with zero mean and covariance matrix $\mathbf{R}$; the vector function $f_d(\cdot)$ is obtained from the state transition function $f(\cdot)$. The augmented state vector is expressed as

$$\mathbf{z}_k^a = [(\mathbf{z}_k)^T (\mathbf{w}_k)^T (\mathbf{v}_k)^T]^T \tag{12}$$

Suppose that the initial state $\mathbf{z}_0$ is a Gaussian noise with mean $\overline{\mathbf{z}}_0$ and covariance matrix $\mathbf{P}_0$. The mean and covariance matrix of the initial state $\mathbf{z}_0^a$ are expressed as follows

$$\overline{\mathbf{z}}_0^a = E[\mathbf{z}_0^a] = [(\overline{\mathbf{z}}_0)^T \quad \mathbf{0} \quad \mathbf{0}]^T \tag{13}$$

$$\mathbf{P}_0^a = E[(\mathbf{z}_0^a - \overline{\mathbf{z}}_0^a)(\mathbf{z}_0^a - \overline{\mathbf{z}}_0^a)^T] = \begin{bmatrix} \mathbf{P_0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{R} \end{bmatrix} \tag{14}$$

The unscented transformation method is used to pick sigma points for estimating the statistics of the state and measurement. Assume the augmented state $\mathbf{z}_k^a$ has mean $\overline{\mathbf{z}}_k^a$ and covariance $\mathbf{P}_k^a$. The matrix of sigma points corresponding to the augmented state $\mathbf{z}_k^a$ is denoted as $\mathbf{\Lambda}_k$, whose columns are calculated by

$$(\mathbf{\Lambda}_k)_0 = \overline{\mathbf{z}}_k^a \tag{15a}$$

$$(\mathbf{\Lambda}_k)_i = \overline{\mathbf{z}}_k^a + \left(\sqrt{(L+\lambda)\mathbf{P}_k^a}\right)_i \qquad i = 1, \cdots, L \tag{15b}$$

$$(\mathbf{\Lambda}_k)_i = \overline{\mathbf{z}}_k^a - \left(\sqrt{(L+\lambda)\mathbf{P}_k^a}\right)_i \qquad i = L+1, \cdots, 2L \tag{15c}$$

where $L$ is the dimension of the augmented state; $\left(\sqrt{(L+\lambda)\mathbf{P}_k^a}\right)_i$ is the $i$th column of the matrix square root of $(L+\lambda)\mathbf{P}_k^a$; $\lambda$ is a scaling parameter, which is defined as

$$\lambda = \varepsilon^2 (L+\kappa) - L \tag{16}$$

where $\varepsilon$ is a small positive constant parameter that determines the spread of the sigma points (a typical recommendation is $\varepsilon = 10^{-3}$); $\kappa$ is a secondary scaling parameter that is usually set to 0. The weights associated with the vectors $(\mathbf{\Lambda}_k)_i$ are calculated by

$$W_0^m = \frac{\lambda}{L+\lambda} \tag{17a}$$

$$W_0^c = \frac{\lambda}{L+\lambda} + (1-\varepsilon^2+\tau) \tag{17b}$$

$$W_i^m = W_i^c = \frac{1}{2(L+\lambda)} \qquad i = 1,\cdots,2L \tag{17c}$$

where $W_i^m$ is the weights for the mean; $W_i^c$ is the weights for the covariance; $\tau$ is a parameter used to incorporate prior knowledge of the distribution of the augmented state, and $\tau$ = 2 is an optimal value for Gaussian noises. The components of sigma points corresponding to the original state $\mathbf{z}_k$, process noise $\mathbf{w}_k$, and measurement noise $\mathbf{v}_k$ are denoted as $\mathbf{\Lambda}_k^z$, $\mathbf{\Lambda}_k^w$, and $\mathbf{\Lambda}_k^v$, i.e., $\mathbf{\Lambda}_k = [(\mathbf{\Lambda}_k^z)^T\,(\mathbf{\Lambda}_k^w)^T\,(\mathbf{\Lambda}_k^v)^T]^T$ .

Subsequently, the sigma points are propagated through the nonlinear state transition function as

$$\mathbf{\Lambda}_{k+1|k}^z = f_d(\mathbf{\Lambda}_k^z, \mathbf{\Lambda}_k^w) \tag{18}$$

The predicted state mean $\bar{\mathbf{z}}_{k+1}^0$ and its covariance $\mathbf{P}_{k+1}^0$ are calculated using the projected sigma points $\mathbf{\Lambda}_{k+1|k}^z$ as

$$\bar{\mathbf{z}}_{k+1}^0 = \sum_{i=0}^{2L} W_i^m (\mathbf{\Lambda}_{k+1|k}^z)_i \tag{19}$$

$$\mathbf{P}_{k+1}^0 = \sum_{i=0}^{2L} W_i^c [(\mathbf{\Lambda}_{k+1|k}^z)_i - \bar{\mathbf{z}}_{k+1}^0][(\mathbf{\Lambda}_{k+1|k}^z)_i - \bar{\mathbf{z}}_{k+1}^0]^T \tag{20}$$

Then, the projected sigma points are substituted into the observation equation to predict the measurement

$$\mathbf{Y}_{k+1|k} = h(\mathbf{\Lambda}_{k+1|k}^z, \mathbf{\Lambda}_k^v) \tag{21}$$

from which the mean and covariance of the predicted measurement, and the cross-covariance of the state and measurement are computed as

$$\bar{\mathbf{y}}_{k+1} = \sum_{i=0}^{2L} W_i^m (\mathbf{Y}_{k+1|k})_i \tag{22}$$

$$\mathbf{P}_{k+1}^{yy} = \sum_{i=0}^{2L} W_i^c [(\mathbf{Y}_{k+1|k})_i - \bar{\mathbf{y}}_{k+1}][(\mathbf{Y}_{k+1|k})_i - \bar{\mathbf{y}}_{k+1}]^T \tag{23}$$

$$\mathbf{P}_{k+1}^{zy} = \sum_{i=0}^{2L} W_i^c [(\mathbf{\Lambda}_{k+1|k}^z)_i - \bar{\mathbf{z}}_{k+1}^0][(\mathbf{Y}_{k+1|k})_i - \bar{\mathbf{y}}_{k+1}]^T \tag{24}$$

The Kalman gain is computed as follows

$$\boldsymbol{\theta}_{k+1} = \mathbf{P}_{k+1}^{zy} (\mathbf{P}_{k+1}^{yy})^{-1} \tag{25}$$

Finally, the updated state mean $\bar{\mathbf{z}}_{k+1}$ and its covariance $\mathbf{P}_{k+1}$ are calculated as follows

$$\bar{\mathbf{z}}_{k+1} = \bar{\mathbf{z}}_{k+1}^0 + \boldsymbol{\theta}_{k+1}(\mathbf{y}_{k+1} - \bar{\mathbf{y}}_{k+1}) \tag{26}$$

$$\mathbf{P}_{k+1} = \mathbf{P}_{k+1}^{0} - \boldsymbol{\theta}_{k+1}\mathbf{P}_{k+1}^{yy}\boldsymbol{\theta}_{k+1}^{T} \tag{27}$$

The augmented UKF is a recursive estimator that mainly comprises the prediction and update phases. In the prediction phase, the mean and covariance of the state are calculated from the state at the previous time step using the unscented transformation. In the update phase, the predicted state is combined with the observation information to refine the state estimate. The *a priori* estimate of the state is updated based on the residual between the measured and predicted responses of the system scaled by the Kalman gain. The updated estimate is termed the *a posteriori* state estimate. The augmented UKF is a powerful nonlinear state estimation tool and it has been shown to be a robust method for high-order nonlinearities of complex systems.

### 2.3 Interstory drift calculation

The displacement responses **x**(t) of the system are directly obtained from the estimated state **z**(*t*). Interstory drifts are calculated from the obtained displacement responses **x**(t) as follows

$$\begin{cases} Y_i(t) = x_i(t) - x_{i-1}(t) & (i > 1) \\ Y_i(t) = x_i(t) & (i = 1) \end{cases} \tag{28}$$

where $Y_i(t)$ denotes the time histories of interstory drift of the *i*th story; $x_i(t)$ is the time histories of displacement responses of the *i*th story.

## 3 NUMERICAL EXAMPLE

Nonlinear responses of a four-story hysteretic shear building model subject to an earthquake motion recorded in the 1989 Loma Prieta earthquake are simulated to verify the effectiveness of the presented method. The hysteretic restoring force in each story of the shear building model is simulated by the Bouc-Wen model as follows

$$f_r(t) = \alpha k q(t) + (1-\alpha) u_y k r(t) \tag{29}$$

$$\dot{r}(t) = \dot{q}(t) - \beta |\dot{q}(t)| |r(t)|^{v-1} r(t) - \eta \dot{q}(t) |r(t)|^{v} \tag{30}$$

where $\alpha$ = ratio of post-yield and pre-yield stiffness, $k$ = initial stiffness, $q(t)$ = interstory drift, $u_y$ = yield displacement, $r(t)$ = normalized hysteretic force, $\beta$ and $\eta$ control the shape of hysteresis loops, and $v$ governs the smoothness of the transition from elastic to plastic response. The lumped mass of each story is 4 kg, and the initial stiffness of each story is 1000 N/m. The yield displacement $u_y$ is 0.3 m for all stories. The damping in the model is 10% damping ratio in all modes. Parameters $\alpha$ = 0.05, $\beta$ = 0.5, $\eta$ = 0.5, $v$ = 1 are chosen in the Bouc-Wen model in the simulation. The duration of the earthquake record is 25 seconds and its sampling frequency is 100 Hz. The responses of displacement, velocity, and acceleration are calculated using the Runge-Kutta fourth-order method. Figure 1 shows the hysteretic loops of all stories, in which the first and second stories behave in strong nonlinearity under seismic loading.

In state estimation, the initial values of the state are $\mathbf{x}_0 = \mathbf{0}$, $\dot{\mathbf{x}}_0 = \mathbf{0}$, $a_0$ = 1.5, $(\delta_i)_0$ = 0.75 ($i$ = 1, 2, …, 4). The absolute acceleration responses of the first and fourth stories are used as the measurement data. A white noise process with 3% RMS noise-to-signal is superimposed to the simulated acceleration responses. Figure 2 shows the estimated and calculated interstory drifts of all stories. The estimated and calculated drift time histories match well. Compared to the calculated maximum interstory drifts, the estimated maximum interstory drifts of the first to fourth stories have the differences of 15.4%, 2.9%, 18.1% and 13.1% respectively. This finding indicates that the presented method is effective in estimating seismic interstory drifts of building structures using a limited number of measured accelerations. Figure 3 shows the variations of the coefficient *a* and reduction factors $\delta_i$ ($i$ = 1, 2, …, 4) during the earthquake. Coefficient *a* does not remain constant during seismic loading and its largest change is about 6.7%. The reduction factors for four stories vary between 0.5 and 0.9.

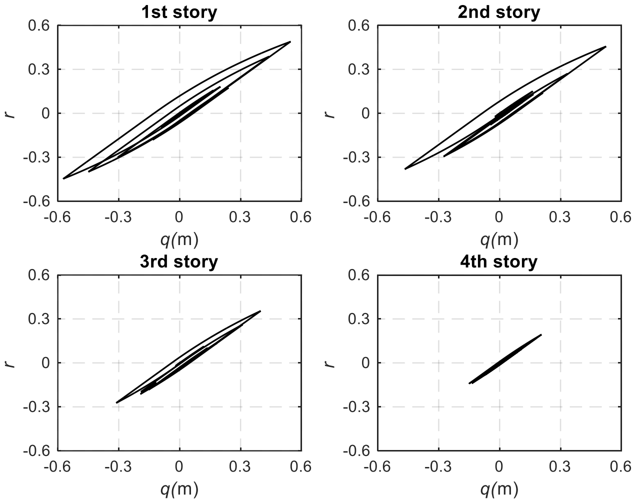


Figure 1. Hysteretic loops of all stories.

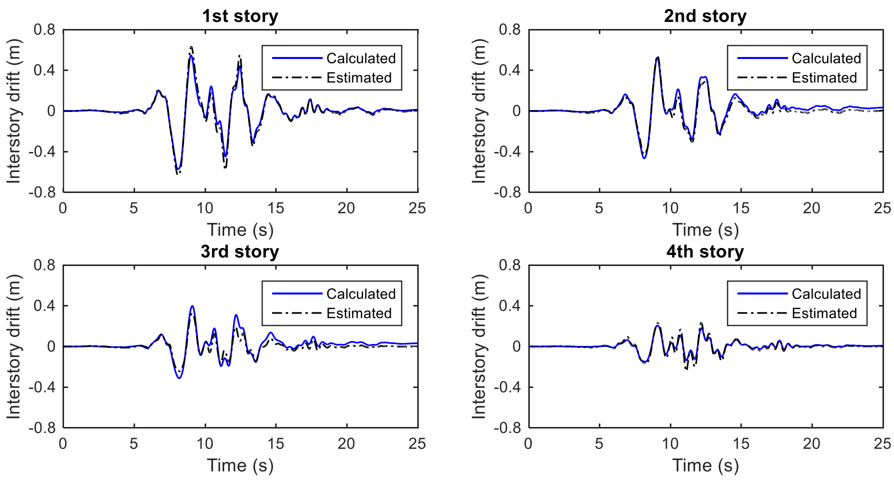


Figure 2. Estimated and calculated interstory drifts.

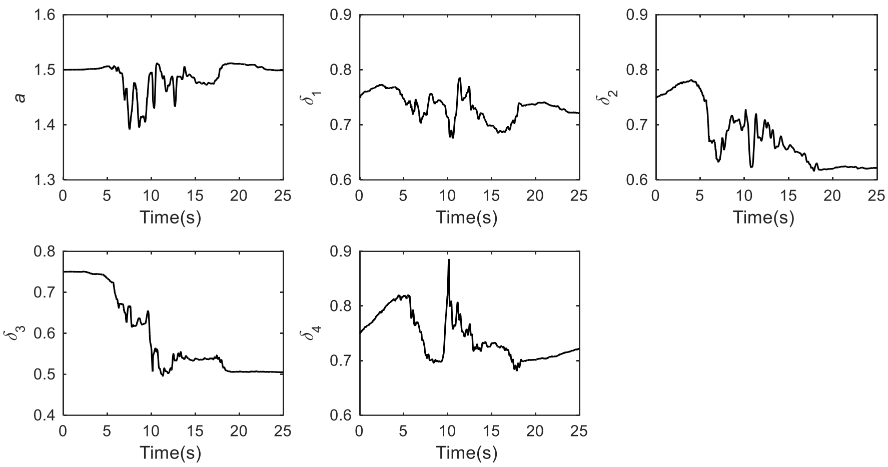


Figure 3. Estimated time-variant coefficient $a$ and reduction factor $\delta_i$ ($i$ = 1, 2, …, 4).

## 4 CONCLUSIONS

This paper presented a time-varying shear model-based method for estimating seismic drift responses of building structures using a limited number of acceleration measurements. The method was derived based on state estimation of a time-varying shear building model with the unscented Kalman filter. The effectiveness of the presented method was numerically verified through a four-story hysteretic shear building model subject to an earthquake motion recorded in the 1989 Loma Prieta earthquake. In the numerical example, the estimated and real interstory drift time histories were nearly identical.

## 5 ACKNOWLEDGMENTS

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